Financial Frictions, Wealth Distribution, and Asset Bubbles

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In this paper, we analyze the relationship between asset bubbles and inequality. The main message are the followings.

- Bubbles increase wealth inequality between entrepreneurs (speculators) and workers (non-speculators).
- As long as bubble size is relatively small, bubbles crowd in productive investments and increase production of an economy. However, once the size becomes too large, then bubbles crowd out even productive investments and reduce production of the economy. In other words, the relationship between bubble size and production is non-monotonic.
- This non-monotonic relationship suggests that, as long as the bubble size is relatively small, both entrepreneurs' wealth and workers' wealth increase. In this region, a trickle-down effect works. We should mention that inequality is enlarged even in this region.
- However, once the size becomes too large sustained by, for example, generous government guarantees, workers' wage income decreases, while entrepreneurs' wealth still increase because of wealth effect of bubbles, leading to increased inequality between workers and entrepreneurs. The trickle-down effect does not work.
- Wealth/income ratio is increased by bubbles.

1 The Model

1.1 Framework

Consider a discrete-time economy with one homogeneous good and a continuum of entrepreneurs and workers. A typical entrepreneur and a representative worker have the following expected discounted utility,

$$E_0\left[\sum_{t=0}^{\infty}\beta^t \log c_t^i\right], \tag{1}$$

where i is the index for each entrepreneur, and c_t^i is the consumption of him/her at date t. $\beta \in (0,1)$ is the subjective discount factor, and $E_0[a]$ is the expected value of a conditional on information at date 0.

Let us start with the entrepreneurs. At each date, each entrepreneur meets high-productivity investment projects (hereinafter H-projects) with probability p, and low productivity ones (L-projects) with probability $1-p$. The investment projects produce capital. The investment technologies are as follows:

$$k_{t+1}^i = \alpha_t^i z_t^i, \tag{2}$$

where $z_t^i (\geq 0)$ is the investment level at date t, and k_{t+1}^i is the capital at date $t+1$ produced by the investment. α_t^i is the marginal productivity of investment at date t. $\alpha_t^i = \alpha^H$ if the entrepreneur has H-projects, and $\alpha_t^i = \alpha^L$ if he/she has L-projects. We assume $\alpha^H > \alpha^L$. For simplicity, we assume that capital fully depreciates in one period.[1] The probability p is exogenous, and independent across entrepreneurs and over time. The entrepreneur knows his/her own type at date t, whether he/she has H-projects or L-projects. Assuming that the initial population measure of each type is p and $1-p$ at date 0, the population measure of each type after date 1 is p and $1-p$, respectively. Throughout this paper, we call the entrepreneurs with H-projects "H-types" and the entrepreneurs with L-projects "L-types".

We assume that because of frictions in a financial market, the entrepreneur can pledge at most a fraction θ of the future return from his/her investment to creditors as in Kiyotaki and Moore (1997). In such a situation, in order for debt contracts to be credible, debt repayment cannot exceed the pledgeable value. That is, the borrowing constraint becomes:

$$r_t b_t^i \leq \theta q_{t+1} \alpha_t^i z_t^i, \tag{3}$$

where q_{t+1} is the relative price of capital to consumption goods at date $t+1$.[2] r_t and b_t^i are the gross interest rate and the amount of borrowing at date t. The parameter $\theta \in (0,1]$, which is assumed to be exogenous, can be naturally taken to be the degree of imperfection of the financial market.

In this economy, there are bubble assets denoted by x. The aggregate

[1] As in Kocherlakota (2009), we can consider a case where only a fraction η of capital depreciates, and consumption goods can be converted one-for-one into capital, and vice-versa. In this setting, we can also obtain the same results as in the present paper.

[2] On an equilibrium path, q_{t+1} is not affected by the collapse of bubbles. Hence, there is no uncertainty with regard to q_{t+1}.

supply of bubble assets is assumed to be constant over time X. As in Tirole (1985), we define bubble assets as those assets that produce no real return, i.e., the fundamental value of the assets is zero. However, under some conditions, the prices of bubble assets become positive, which means that bubbles arise in equilibrium. Here, following Weil (1987), we consider stochastic bubbles, in the sense that they may collapse. In each period, bubble prices become zero (i.e., bubbles burst) at a probability of $1-\pi$ conditional on survival in the previous period. When π is lower, the bursting probability is higher. Once bubbles collapse, they do not arise again unless agents change their expectations about their formation through, for example, unexpected shocks. Let P_t be the per unit price of bubble assets at date t on survival in terms of consumption goods.

The entrepreneur's flow of funds constraint is given by

$$c_t^i + z_t^i + P_t x_t^i = q_t \alpha_{t-1}^i z_{t-1}^i - r_{t-1} b_{t-1}^i + b_t^i + P_t x_{t-1}^i. \tag{4}$$

where x_t^i is the level of bubble assets purchased by a type i entrepreneur at date t. The left hand side of (4) is expenditure on consumption, investment, and the purchase of bubble assets. The right hand side is the available funds at date t, which is the return from investment in the previous period minus debts repayment, plus new borrowing, the return from selling bubble assets. We define the net worth of the entrepreneur at date t as $e_t^i \equiv q_t \alpha_{t-1}^i z_{t-1}^i - r_{t-1} b_{t-1}^i + P_t x_{t-1}^i$.

We also impose the short sale constraint on bubble assets:

$$x_t^i \geq 0. \tag{5}$$

Let us now turn to the maximization problem of workers. There are workers with a unit measure. Each worker is endowed with one unit of labor endowment in each period, which is supplied inelastically in labor markets, and earns wage income, w_t. Workers do not have investment opportunities, and cannot borrow against their future labor incomes. The flow of funds constraint, the borrowing constraint, and the short sale constraint for them are given by

$$c_t^u + P_t x_t^u = w_t - r_{t-1} b_{t-1}^u + b_t^u + P_t x_{t-1}^u, \tag{6}$$

$$r_t b_t^u \leq 0, \tag{7}$$

$$x_t^u \geq 0, \tag{8}$$

where ^{u} represents workers.

Lastly, we explain the production technology. There are competitive firms which produce final consumption goods using capital and labor. The production function of each firm is

$$y_t = k_t^\sigma n_t^{1-\sigma}, \tag{9}$$

where y_t is output of each firm. k_t and n_t are capital and labor input, respectively.

Factors of production are paid their marginal product:

$$q_t = \sigma K_t^{\sigma-1} \quad \text{and} \quad w_t = (1-\sigma)K_t^{\sigma}, \tag{10}$$

where K_t is the aggregate capital stock at date t.

1.2 Equilibrium

Let us denote the aggregate consumption of H-and L-types and workers at date t as $\sum_{i\in H_t} c_t^i \equiv C_t^H$, $\sum_{i\in L_t} c_t^i \equiv C_t^L$, C_t^u, where H_t and L_t mean a family of H-and L-types at date t. Similarly, let Y_t, $\sum_{i\in H_t} z_t^i \equiv Z_t^H$, $\sum_{i\in L_t} z_t^i \equiv Z_t^L$, $\sum_{i\in H_t} b_t^i \equiv B_t^H$, $\sum_{i\in L_t} b_t^i \equiv B_t^L$, B_t^u, $\sum n_t \equiv N_t$, $(\sum_{i\in H_t\cup L_t} x_t^i + X_t^u) \equiv X_t$ be the aggregate output, the aggregate investments of each type, the aggregate borrowing of each type, the aggregate labor input, and the aggregate demand for bubble assets. Then, the market clearing condition for goods, credit, capital, labor, and bubble assets are

$$C_t^H + C_t^L + C_t^u + Z_t^H + Z_t^L = Y_t, \tag{11}$$

$$B_t^H + B_t^L + B_t^u = 0, \tag{12}$$

$$K_t = \sum_{i\in H_t\cup L_t} k_t^i, \tag{13}$$

$$N_t = 1, \tag{14}$$

$$\sum_{i \in H_t \cup L_t} (x_t^i + x_t^u) = X. \quad (15)$$

The competitive equilibrium is defined as a set of prices $\{r_t, w_t, P_t, q_t\}_{t=0}^{\infty}$ and quantities $\left\{C_t^H, C_t^L, C_t^u, B_t^H, B_t^L, B_t^u, Z_t^H, Z_t^L, X_t, N_t, K_{t+1}, Y_t, N_t\right\}_{t=0}^{\infty}$, such that (i) the market clearing conditions, (11)-(15), are satisfied in each period, and (ii) each entrepreneur chooses consumption, borrowing, investment, and the amount of bubble assets, $\{c_t^i, b_t^i, z_t^i, x_t^i\}_{t=0}^{\infty}$, to maximize his/her expected discounted utility (1) under the constraints (2)-(5), and (iii) each worker chooses consumption, borrowing, and the amount of bubble assets, $\{c_t^u, b_t^u, x_t^u\}_{t=0}^{\infty}$, to maximize his/her expected discounted utility (1) under the constraints (6)-(8).

1.3 Optimal Behavior of Entrepreneurs and Workers

We now characterize the equilibrium behavior of entrepreneurs and workers in the bubble economy. We focus on the equilibrium where

$$q_{t+1}\alpha^L \leq r_t < q_{t+1}\alpha^H.$$

In equilibrium, interest rate must be at least as high as $q_{t+1}\alpha^L$, since nobody lends to the projects if $r_t < q_{t+1}\alpha^L$. Moreover, if the interest rate is higher than the rate of return of H-projects, nobody borrows.

Since the utility function is log-linear, each entrepreneur consumes a fraction $1-\beta$ of the net worth in each period, that is, $c_t^i = (1-\beta)e_t^i$. For H-types at date t, the borrowing constraint (3) is binding since $r_t < q_{t+1}\alpha^H/(1-\tau_t^s)$ and the investment in bubbles is not attractive, that is, (5) is also binding. We will verify this result in the Technical Appendix ?. Then, by using (3), (4), and (5), the investment function of H-types at date t can be written as

$$z_t^i = \frac{\beta(q_t\alpha_{t-1}^i z_{t-1}^i - r_{t-1}b_{t-1}^i + P_t x_{t-1}^i)}{1 - \dfrac{\theta q_{t+1}\alpha^H}{r_t}}. \tag{16}$$

From this investment function, we understand that for the entrepreneurs who purchased bubble assets in the previous period, they are able to sell those assets at the time they encounter H-projects. As a result, their net worth increases, which boosts their investments. That is, as Hirano and Yanagawa (2010) showed, bubbles generate a crowd-in effect on productive investments. In our model, the entrepreneurs buy bubble assets for speculative purpose. They buy bubble assets when they have L-projects, and sell those assets when they have opportunities to invest in H-projects.

For L-types at date t, since $c_t^i = (1-\beta)e_t^i$, the budget constraint (4) becomes

$$z_t^i + P_t x_t^i + (-b_t^i) = \beta e_t^i. \tag{17}$$

Each L-type allocates his/her savings, βe_t^i, into three assets, i.e., z_t^i, $P_t x_t^i$, and $(-b_t^i)$. Each L-type chooses optimal amounts of b_t^i, x_t^i, and z_t^i so that the expected marginal utility from investing in three assets is equalized. By solving the utility maximization problem explained in the Technical Appendix ?, we can derive the demand function for bubble assets of a L-type:

$$P_t x_t^i = \frac{\pi \frac{P_{t+1}}{P_t} - r_t}{\frac{P_{t+1}}{P_t} - r_t} \beta e_t^i, \tag{18}$$

The remaining fraction of savings is split across z_t^i and $(-b_t^i)$:

$$z_t^i + (-b_t^i) = \frac{(1-\pi)\frac{P_{t+1}}{P_t}}{\frac{P_{t+1}}{P_t} - r_t} \beta e_t^i.$$

Since investing in L-projects (z_t^i) and secured lending to other entrepreneurs $(-b_t^i)$ are both safe assets, $z_t^i \geq 0$ if $r_t = q_{t+1}\alpha^L$, and $z_t^i = 0$ if $r_t > q_{t+1}\alpha^L$. That is, the following conditions must be satisfied:

$$(r_t - q_{t+1}\alpha^L) z_t^i = 0, \; z_t^i \geq 0, \text{ and } r_t - q_{t+1}\alpha^L \geq 0.$$

Moreover, when $r_t = q_{t+1}\alpha^L$, investing in L-projects and secured lending to other entrepreneurs are indifferent for L-types, aggregate investment level of L-types, Z_t^L, is determined from (11).

Next, regarding the optimal behavior of workers, we can show that workers do not save in equilibrium (see the Technical Appendix for the proof.) That is,

$$c_t^u = w_t.$$

1.4 Dynamics

(11) can be written as

$$Z_t^H + Z_t^L + P_t X = \beta A_t. \tag{19}$$

Then, we have the evolution of aggregate capital stock:

$$K_{t+1} = \begin{cases} \alpha^H \dfrac{\beta p A_t}{1 - \frac{\theta \alpha^H}{\alpha^L}} + \alpha^L \left(\beta A_t - \dfrac{\beta p A_t}{1 - \frac{\theta \alpha^H}{\alpha^L}} - P_t X \right) & \text{if } r_t = q_{t+1}\alpha^L, \\ \\ \alpha^H \left[\beta A_t - P_t X \right] & \text{if } r_t > q_{t+1}\alpha^L. \end{cases} \tag{20}$$

where $A_t \equiv q_t K_t + P_t X$ is the aggregate wealth of entrepreneurs at date t, and $\sum_{i \in H_t} e_t^i \equiv pA_t$ is the aggregate wealth of H-types at date t. (More details about aggregation of each variable will be explained in the Technical Appendix ?). When $r_t = q_{t+1}\alpha^L$, both H-and L-types may invest. The first term and the second term of the first line represent the capital stock at date $t+1$ produced by H-and L-types, respectively. When $r_t > q_{t+1}\alpha^L$, only H-types invest. From (19), we know $Z_t^H = \beta A_t - P_t X$. $(-P_t X)$ in (20) captures a traditional crowd-out effect of bubbles analyzed in Tirole (1985), i.e., the presence of bubble assets crowds savings away from investments.

As long as $r_t \geq q_{t+1}\alpha^L$, the interest rate is determined by the credit market clearing condition (12), which can be written as

$$\frac{\beta p A_t}{1 - \frac{\theta q_{t+1}\alpha^H}{r_t}} + P_t X = \beta A_t.$$

That is, the aggregate savings of entrepreneurs, βA_t, flow to aggregate H-investments and bubbles. By defining $\phi_t \equiv P_t X / \beta A_t$ as the size of bubbles, we can rewrite the above relation as

$$r_t = \frac{q_{t+1}\theta\alpha^H(1-\phi_t)}{1-p-\phi_t}.$$

It follows that r_t increases with ϕ_t, reflecting the tightness of the credit markets.

Thus, the equilibrium interest rate is determined as

$$r_t = q_{t+1} Max\left[\alpha^L, \frac{\theta\alpha^H(1-\phi_t)}{1-p-\phi_t}\right]. \tag{21}$$

In other words, $r_t = q_{t+1}\alpha^L$ if $\phi_t < \phi^*$, and $r_t = \frac{\theta\alpha^H(1-\phi_t)}{1-p-\phi_t}$ if $\phi_t \geq \phi^*$, where $\phi^* \equiv \frac{\alpha^L(1-p)-\theta\alpha^H}{\alpha^L-\theta\alpha^H}$.

2 Dynamics of Rational Bubbles

Next, we examine the dynamics of rational bubbles. Since we assume that rational bubbles are stochastic, that is, bubbles persist with probability $\pi(< 1)$, here, we focus on the dynamics of bubbles until bubbles collapse.

From the definition of $\phi_t \equiv P_t X / \beta A_t$, ϕ_t evolves over time as

$$\phi_{t+1} = \frac{\frac{P_{t+1}}{P_t}}{\frac{A_{t+1}}{A_t}} \phi_t. \tag{22}$$

The evolution of the size of bubbles depends on the relation between the growth rate of wealth and the growth rate of bubbles. When we aggregate (18), and solve for P_{t+1}/P_t, then we obtain the required rate of return on bubble assets:

$$\frac{P_{t+1}}{P_t} = \frac{r_t(1-p-\phi_t)}{\pi(1-p)-\phi_t}, \tag{23}$$

where $(1-p-\phi_t)/[\pi(1-p)-\phi_t]$ is the risk premium on bubble assets.

By using (21), (23), the definition of aggregate wealth of entrepreneurs, (22) can be written as

$$\phi_{t+1} = \begin{cases} \dfrac{\dfrac{(1-p-\phi_t)}{\pi(1-p)-\phi_t}}{\left(1+\dfrac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta + \dfrac{(1-\pi)(1-p)}{\pi(1-p)-\phi_t}\beta\phi_t}\phi_t & \text{if } \phi_t < \phi^*, \\ \\ \dfrac{\theta}{\beta}\dfrac{1}{\pi(1-p)-(1-\theta)\phi_t}\phi_t & \text{if } \phi_t \geq \phi^*. \end{cases} \tag{24}$$

Using this (24), we examine the sustainable dynamics of ϕ_t. In order for bubbles to be sustainable, the following condition must be satisfied for any t:

$$\phi_t < 1. \tag{25}$$

Violation of this condition means explosion of bubbles.

As examined in the literature (Tirole 1985; Weil 1989; Farhi and Tirole 2012b), dynamics of bubbles take three patterns. The first one is that bubbles become too large and explode to $\phi_t \geq 1$. This dynamic path cannot be sustained by this economy and thus, bubbles cannot exist in this pattern. The second pattern is that ϕ_t becomes smaller over time and converges to zero. This path is called asymptotically bubbleless. In this dynamic path, the effects of bubbles converge to zero. Hence, we exclude this path from our consideration as usual in the literature. The third pattern is that ϕ_t converges to a positive value as long as the bubbles survive. This dynamic path is a saddle point path where the economy converges to a stochastic stationary-state with positive bubbles as long as bubbles persist.[3] In this paper, we concentrate on this saddle path equilibrium as usual in the literature (Tirole 1985; Weil 1989; Farhi and Tirole 2012b).

By using ϕ_t (20) can be written as

$$K_{t+1} = \begin{cases} \dfrac{\left[(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p)\beta\alpha^L - \alpha^L\beta\phi_t\right]}{1-\beta\phi_t}\sigma K_t^{\sigma} & \text{if } \phi_t < \phi^*, \\ \\ \dfrac{\alpha^H\beta\left[1-\phi_t\right]}{1-\beta\phi_t}\sigma K_t^{\sigma} & \text{if } \phi_t \geq \phi^*. \end{cases} \tag{26}$$

As long as bubbles can exist (We explain the existence condition of bubbles in

[3]Definition of stochastic stationary state follows Weil (1989). In the stochastic stationary state, all variables $(K_t, A_t, q_t, r_t, w_t, P_t, \phi_t)$ become constant over time as long as bubbles persist.

Proposition 1), the dynamics of $K_{t+1}/K_t^\sigma = K_{t+1}/Y_t$ is an increasing function of ϕ_t as long as $\phi_t < \phi^*$, and it becomes a decreasing function of ϕ_t if $\phi_t \geq \phi^*$. Intuitively, as long as the size of bubbles is small, i.e., $\phi_t < \phi^*$, both H-and L-types invest. An increase in the size of bubbles crowds L-projects out and crowds in H-projects, thus increasing capital stock. In $\phi_t < \phi^*$, the crowd-in effect dominates the crowd-out effect at the margin. We call this region "under-sized bubble" region. When the size of bubbles equals $\phi_t = \phi^*$, all L-projects are completely crowded out, and only H-types invest if $\phi_t \geq \phi^*$. If the size of bubbles becomes larger in $\phi_t > \phi^*$, even H-projects are crowded out, thus reducing capital stock. Overly large size bubble, i.e., an excessive speculation in bubble assets, is harmful to production. In $\phi_t > \phi^*$, the crowd-out effect dominates the crowd-in effect at the margin. Thus, we call this region "over-sized bubble" region. As Hirano et al. (2013) showed, the relationship between bubble size and capital stock (output) is non-monotonic as in Figure 1. The following Proposition summarizes this.[4]

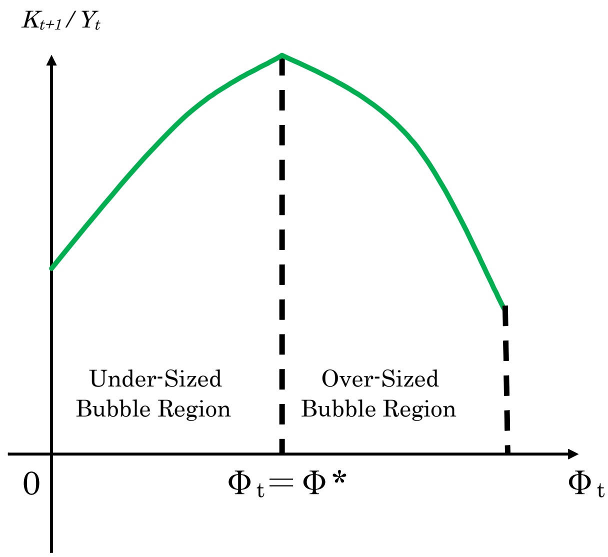
Proposition 1 *Relationship between Bubble Size and Capital Stock (Output): K_{t+1}/K_t^σ is an increasing function of ϕ_t as long as $\phi_t \leq \phi^*$, and it becomes a decreasing function of ϕ_t if $\phi_t > \phi^*$. There is a size of bubbles $\phi^* \equiv \frac{\alpha^L(1-p)-\theta\alpha^H}{\alpha^L-\theta\alpha^H}$ maximizes the capital stock and output for any t (i.e., not*

[4] It is well known that theoretically bubble assets and government bonds have the similar property. Therefore, Proposition 1 also means that the relationship between government bonds-to-GDP ratio and output (or output growth rate if the model is based on an endogenous growth rate) is non-monotonic. In other words, as long as the debt-to-GDP ratio is relatively small, government debt enhances output (or output growth rate), but once the ratio becomes too large, then it reduces output (or output growth rate).

just before the bursts, but also after the bubble bursts).

The dynamic system of this economy is characterized by (26) and (24). However, (24) is independent from K_t and the dynamics of ϕ_t is derived only by (24). From (24), we can derive that ϕ_t must be constant over time unless ϕ_t is asymptotically bubbleless. This means that on the saddle path equilibrium, wealth of entrepreneurs and bubbles grow at the same rate.

Bubble Size and Production



3 Effects of Asset Bubbles

3.1 Existence of Asset Bubbles

We first characterize the existence condition of stochastic bubbles. In other words, we investigate whether a dynamic path with bubbles does not explode. Mathematically, we check whether the dynamic system (24) has a non-negative steady-state, $\phi_t = \phi$. As we show below, the financial market condition, θ, is crucial to the existence condition of bubbles. (Hereafter, proofs of all Propositions are given in Appendix).

Proposition 2 *Stochastic bubbles with survival probability* π *can exist if and only if* θ *satisfies the following condition,*

$$\underline{\theta} \equiv \max\left[\frac{\alpha^L - \pi\beta[\alpha^L + (\alpha^H - \alpha^L)p]}{\alpha^H(1-\pi\beta)}, 0\right] < \theta < \overline{\theta} \equiv \pi\beta(1-p).$$

From this Proposition 2, we can understand that bubbles tend to exist when the degree of financial imperfection, θ, is in the middle range. In other words, improving financial market conditions might enhance the possibility of bubbles when the initial condition of θ is low. This result is similar to the result in Hirano and Yanagawa (2010) who characterize the existence condition of bubbles in an endogenous growth framework. We can obtain a similar result even in a non-growth model. Intuitively, if θ is low, enough resources cannot be transferred to productive sector, because the borrowing constraint is sufficiently tight. As a result, economic growth rate with bubbles becomes low. On the other hand, interest rate can not be lower than the rate of return on L-projects and bubbles grow faster than interest rate. As a result, under a very low θ, bubbles' growth rate becomes higher than the economic growth rate. Hence, the bubbles are unlikely to arise under a very low θ.

3.2 Under-Sized Bubbles and Over-Sized Bubbles

Moreover, within the bubble regions, there are two different regions, i.e., under-sized bubble region and over-sized bubble region. As Hirano et al. (2013) showed, the relation between bubble size and capital stock is non-monotonic like Figure 1. In other words, there is the bubble size, $\phi = \phi^*$, that maximizes capital stock and output for any t. Equilibrium bubble size on the saddle path may be larger or smaller than ϕ^*. More specifically, when we solve for equilibrium bubble size without government policy, we have

$$\phi(\theta) = \begin{cases} \dfrac{\pi - \dfrac{1-\pi\beta(1-p)}{\left[1+(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H})p\right]\beta - \beta(1-p)}}{1 - \dfrac{1-\pi\beta(1-p)}{\left[1+(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H})p\right]\beta - \beta(1-p)}}(1-p) < \phi^* & \text{if } \underline{\theta} < \theta < \theta^*, \\ \\ \dfrac{\pi\beta(1-p)-\theta}{\beta(1-\theta)} > \phi^* & \text{if } \theta^* < \theta < \overline{\theta}, \end{cases} \tag{27}$$

I.e, equilibrium bubble size depends on the quality of the financial system, where θ^* is given in Appendix. We summarize (27) in Proposition 3.

Proposition 3 *(i) If $\underline{\theta} < \theta < \theta^*$, equilibrium bubble size is smaller than ϕ^*. In other words, within the bubble regions and if the quality of the financial system is relatively low, under-sized bubbles arise.*

(ii) If $\theta^* < \theta < \overline{\theta}$, equilibrium bubble size is larger than ϕ^*. In other words, within the bubble regions and if the quality of the financial system is

relatively high, over-sized bubbles arise.

Proposition 2 suggests that in advanced economies where the quality of the financial system tends to be better than that of emerging enomomies, over-sized bubbles are more likely to arise, while in emerging economies, under-sized bubbles are more likely to emerge. Intuitively, within the bubble regions and if θ is relatively high, enough savings can be transferred to H-projects even without bubbles, because the borrowing constraint is relatively loose. In this situation, bubbles crowd out even H-projects at the margin as well as crowd out L-projects completely. On the other hand, within the bubble regions and if θ is relatively low, L-types cannot lend all of their savings to H-types even with bubbles, because the borrowing constraint is sufficiently tight. As a result, L-types hold a lot of idle savings, but they don't want to invest all of their idle savings into bubble assets, because bubbles are risky and they are risk-averse agents. In equilibrium, they end up investing in their own safe projects with low returns for risk-hedge. This means that in aggregate, some of the savings in the economy flow to L-projects.

3.3 Macroeconomic Effects of Asset Bubbles

Together with Proposition 3, we can characterize whether bubbles are expansionary or contractionary in output compared to the bubbleless economy, and how those effects are related to the quality of the financial system. Equilibrium dynamics of capital stock can be derived by substituing (27) into (26).

Proposition 4 *There exists a threshold level of* $\theta = \theta_1$ $(> \theta^*)$*. (i) If* $\underline{\theta} < \theta < \theta^*$*, under-sized bubbles arise and output in the bubble economy is higher than that in the bubbleless economy for any* $t \geq 1$*, given an initial* Y_0*. (ii) If* $\theta^* < \theta < \theta_1$*, over-sized bubbles arise and output in the bubble economy is higher than that in the bubbleless economy for any* $t \geq 1$*, given an initial* Y_0*. (iii) If* $\theta_1 < \theta < \overline{\theta}$*, over-sized bubbles emerge and output in the bubble economy is lower than that in the bubbleless economy for any* $t \geq 1$*, given an initial* Y_0*.*

Figure 2 illustrates Proposition 3 and 4. In Figure 2, we compare output in the stochastic stationary state of the bubble economy with output in the steady-state of the bubbleless economy. In the Appendix, we provide a full characterization of the bubbleless economy and derive θ_1. Proposition 4 shows that in $\underline{\theta} < \theta < \theta_1$, bubbles increase output compared to the bubbleless economy, while in $\theta_1 < \theta < \overline{\theta}$, bubbles decrease output.[5] Intuitively, if the quality of the financial system is relatively low and if there is no bubble, enough savings cannot be transferred to H-types and even L-types end up producing in equilibrium. In this situation, once bubbles arise, bubbles crowd out L-projects and crowds in H-projects, thereby increasing output. On the other hand, if the quality of the financial system is relatively high, even without bubbles, the financial system can allocate enough funds to H-projects. In this situation, bubbles end up crowding out H-projects largely,

[5] Hirano and Yanagawa (2010) provide a full characterization on the relation between bubbles and long-run economic growth rate in an endogenous growth framework.

thus reducing output.

In the rest of our analyses, we restrict our attention to $\underline{\theta} < \theta < \theta_1$.

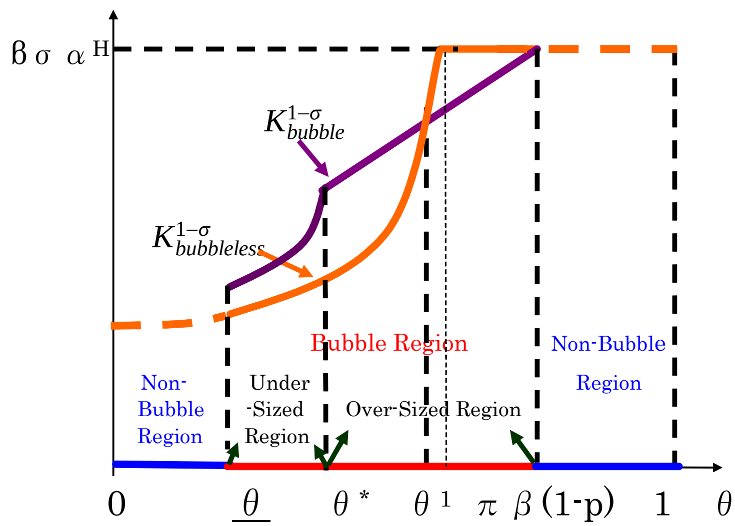


Figure 2: Bubbles and Capital Stock

3.4 Wealth Inequality and Bubbles

When we compute average wealth level of entrepreneurs in the stochastic stationary state of the bubble economy, $(A^e)^{bubble}$, and that in the steady state of the bubbleless economy, $(A^e)^{bubbleless}$, we obtain

$$(A^e)^{bubble} = \sigma K_{bubble}^{\sigma} + PX = \frac{1}{1-\beta\phi}\sigma Y_{bubble} > (A^e)^{bubbleless} = \sigma K_{bubbleless}^{\sigma} = \sigma Y_{bubbleless}. \tag{28}$$

Likewise, we can derive average wealth level of workers, A^u. Here we define wealth of workers as wage income, because workers do not save in equilibrium.

$$(A^u)^{bubble} = (1-\sigma)Y_{bubble} > (A^u)^{bubbleless} = (1-\sigma)Y_{bubbleless}. \tag{29}$$

I.e., average wealth levels of both entrepreneurs and workers are strictly higher in the bubble economy.

Then, from (28) and (29), we obtain

$$\left(\frac{A^e}{A^u}\right)^{bubble} = \frac{\sigma}{1-\sigma}\frac{1}{1-\beta\phi} > \left(\frac{A^e}{A^u}\right)^{bubbleless} = \frac{\sigma}{1-\sigma}. \tag{30}$$

I.e., bubbles lead to increased wealth inequality between entrepreneurs who are bubble holders and workers who are non-bubble holders. For entrepreneurs, their wealth increases more because of wealth effect of bubbles. Moreover, (30) says that the extent of inequality depends on equilibrium bubble size, ϕ, i.e., inequality becomes larger, the larger equilibrium bubble size.

3.5 Bubbles, Trickle-Down Effect, and Inequality

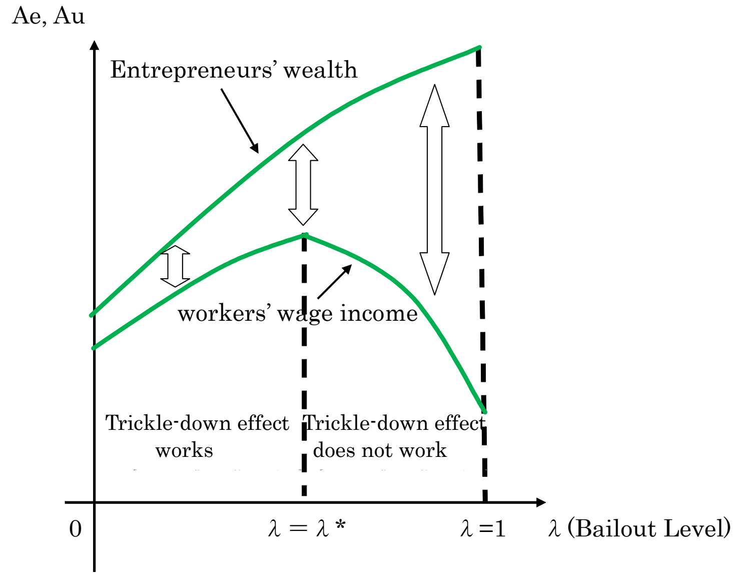
As Hirano et al. (2013) showed, expectations about government guarantees increase equilibrium bubbles size monotonically. In Hirano et al. (2013), when bubbles collapse, to mitigate a free-fall in output and welfare, government bails out entrepreneurs who suffer losses from bubble investments, i.e., government guarantees bubble investments against losses. Given this monotonic relationship, we can derive the following implications.

When under-sized bubbles arise in laissez-faire economy (i.e., in $\underline{\theta} < \theta < \theta^*$), an increase in government guarantess initially enhances production of the economy, leading to increased wealth for everybody (wage income of workers increases and entrepreneurs' wealth increases). Therefore, a trickle-down effect works, even though wealth inequality between entrepreneurs and workers is enlarged. However, once government guarantees are too generous, then over-sized bubbles are created, and even productive investments are crowded out, thereby reducing production level of the economy. As a consequence, workers' wage income decreases, while entrepreneurs' wealth still increases because of wealth effect of bubbles, leading to increased inequality. That is, when over-sized bubbles occur sustained by too generous government guarantees, the trickle-down effect does not work.

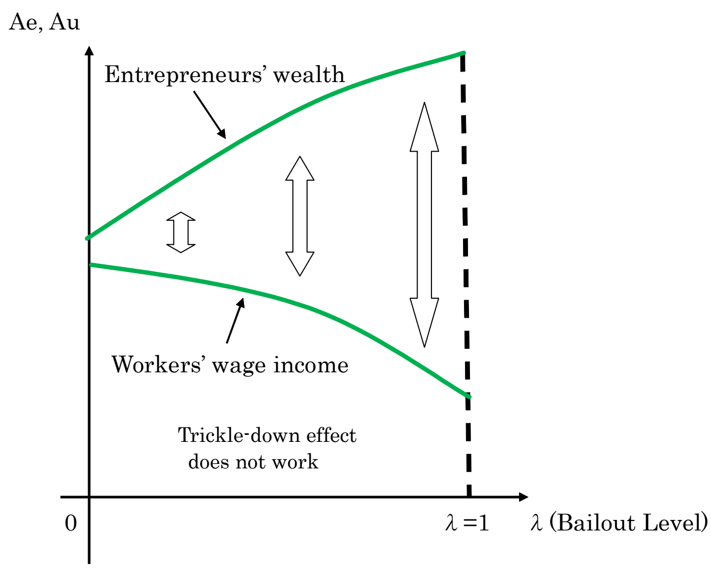
When over-sized bubbles arise in laissez-faire economy (i.e., $\theta^* < \theta < \overline{\theta}$), as mentioned above, the trickle-down effect does not work.

Figure 3 summarizes the above mentioned implications.

When under-sized bubbles occur in laissez-faire economy,



When over-sized bubbles occur in laissez-faire economy,



3.6 Wealth/Income Ratio

When we compute wealth/income ratio, we obtain

$$\left(\frac{A}{Y}\right)^{bubble} = \frac{\sigma}{1-\beta\phi} > \left(\frac{A}{Y}\right)^{bubbleless} = \sigma.$$

I.e., wealth/income ratio is increased by bubbles. Moreover, the ratio is an increasing function of equilibrium bubble size, ϕ.