Uncertainty in Global Sourcing

Learning, sequential offshoring, and selection patterns.

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Abstract

Institutions play an important role shaping the multinational firms' sourcing decisions worldwide. I focus in situations in which firms face uncertainty about institutions abroad. In a two-country model, I characterise a dynamic model with informational spillovers, which allows firms to better assess their offshoring potential by observing other firms behaviours. The equilibrium path adopts a sequential offshoring dynamic, led by the most productive firms in the market. In the long run, the information spillovers drive to economy to the perfect information steady state, vanishing the negative welfare effects produced by the prior uncertainty. However, in a multi-country world, the model leads to a multiple equilibria situation. I show that, in some equilibria, selection patterns emerge in firms' choices about the offshoring locations. In these cases, the informational spillovers become a driver of the sectoral specialisation of countries, and thence a source of their comparative advantages. I also show the long run welfare consequences of the multiple equilibria.

1 Introduction

Many scholars have observed that intermediate inputs explain a substantial share of global trade, revealing a deep transformation of the structure of international trade. It is widely accepted that the declining trade barriers and technological advances in transportation and ICT have encouraged an increasing fragmentation of production across the world. Sourcing strategies have become global, driving and shaping the emergence of regional and global production networks (Helpman, 2006; Grossman and Rossi-Hansberg, 2008; Antràs and Helpman, 2008; Alfaro and Charlton, 2009; Antràs and Yeaple, 2014; Ramondo et al., 2015).

The growing role of multinational firms in the organisation of international trade has aroused the interest of many scholars to understand the underlying determinants of firms' global strategies, particularly in regard to the organisation of value chains on a global scale (Helpman, 2006; Antràs and Helpman, 2004, 2008). Recent literature has focused the attention on how institutions, especially those related to contract enforcement, affect the location of production, the investment decisions, and the firms' optimal sourcing strategies or technology choices (Helpman, 2006; Acemoglu et al., 2007; Antràs and Helpman, 2008; Antràs and Chor, 2013). Other scholars have examined the role of institutions in the definition of the countries' comparative advantages (Costinot, 2009; Acemoglu et al., 2007; Nunn, 2007; Levchenko, 2007).

Decisions taken under perfect information is an extensive feature of many of the models on global sourcing. However, the sourcing decisions of multinational firms are usually taken under uncertainty, particularly about the prevailing conditions in foreign countries, such as infrastructure quality or institutional fundamentals. The vague knowledge that firms posses about locations where they have never been active before may be the more obvious case reflecting the underlying uncertainty of global sourcing decisions[1]. But uncertainty may also emerge with respect to locations where firms have had some experience in the past. For instance, consider the situation in which the government of one of those countries have implemented a deep and ambicious institutional reform with the aim of stimulate FDI investments and/or promote the insertion of local intermediate inputs producers into global production networks. After the announcement and implementation of the institutional changes, multinational firms may still have doubts about the true scope of the reform, thus raising uncertainty about the institutional fundamentals.[2]

In any case, firms are able to learn about those conditions abroad by their own experience, through the interaction with local agents and institutions, or by exploiting informational externalities, via observation of the behaviour of other firms who are active in those locations. The following sections show that uncertainty and informational spillovers are important features of the context in which global sourcing decisions are taken, with profound implications on the dynamics of the offshoring decisions, the sectoral specialisation of countries and welfare.

In section 1.1, I present a brief review of the small but growing recent literature about uncertainty in trade and in global sourcing. This paper differs from most of the literature on uncertainty in global sourcing (Carballo, 2016; Kohler et al., 2018) and uncertainty in trade (Ramondo et al., 2013; Rob and Vettas, 2003; Handley, 2014) in one particular but important aspect. While those models focus the analysis on firms' decisions under a stochastic environment, I study the dynamics of the decision of firms when they are able to reduce the uncertainty by learning, and thus progressively discover their offshoring potential and adjust their optimal sourcing strategies. In this sense, Segura-Cayuela and Vilarrubia (2008); Albornoz et al. (2012); Nguyen (2012); Aeberhardt et al. (2014); Araujo et al. (2016) are the closer approaches within this literature, although they all focus on export deci-

[1]Firms usually have a vague knowledge about institutional quality such as the reliability of the court system, expropiation risk, or complexity of the regulatory or tax systems, above all, about countries where they have never been active.

[2]Another example: After an ambicious public investment plan executed by a foreign government, the multinational firms may have doubts about the real scope or quality of that infrastructure. Thus, it raises uncertainty about the true conditions of the infrastructure supply in that country.

sions. The dissimilar nature of the sourcing with respect to export decisions leads to significantly different results between those models and (the multiple equilibria nature of) this sourcing model.

The primary goal of this paper is to understand how the existence of prior uncertainty about the institutional fundamentals in foreign countries may affect sourcing strategies, particularly the offshoring decision, and comprehend its implications in terms of sectoral specialisation of countries and welfare. For this purpose, the basic specification of the model represents the uncertainty as an ambiguous prior knowledge of the fixed costs of production in foreign countries[3].

To address these questions, I present a model in a two countries framework (North-South), and I extend the analysis later on to a multi-country world economy.

The main result of the model in a world with two countries consists on the characterisation of a dynamic equilibrium path with informational spillovers, which allows firms to learn about their offshoring potential in the foreign location. In consequence, the equilibrium path takes the form of a sequential offshoring process, led by the most productive firms in the market. Furthermore, the convergence properties of the model shows that the initial welfare losses produced by uncertainty vanishes progressively as the model converges to the perfect information steady state of the economy.

Finally, I extend the model to a multi-country world. Under this context, the informational spillovers lead to a multiple equilibria. In some equilibria, clear selection patterns emerge in the multinational firms' decisions about the locations of their suppliers, which differ from the perfect information steady state of the economy. This unravels the underlying factors that may explain why firms may concentrate their offshoring activity in certain countries, although it differs from their optimal allocation of production across countries from a fundamentals perspective. In other words, the multi-country model reveals a situation of multiple equilibria, in which informational spillovers become a source of sectoral specialization and comparative advantages. In this sense, the model complements the mentioned literature on the role of institutions as determinants of countries' comparative advantages (Acemoglu et al., 2007; Costinot, 2009). Furthermore, the extension to a multi-country world shows that the welfare gains from offshoring may not be fully achieved in the long run in some of the possible equilibria.

The multi-country model remarks the importance of uncertainty in firms' global sourcing decisions, and the risks (and costs) involved when new potential sourcing locations are explored[4]. In other words, learning about their offshoring potential across the world is extremely costly and inefficient for multinational firms.

From a policy perspective, when informational frictions are present in the market, improvements in institutional fundamentals may not have the expected re-

[3] Instead, in a future extension of the model, I introduce uncertainty in the degree of contractibility of the investments within a contractual frictions framework as defined by Acemoglu et al. (2007) and Antràs and Helpman (2008).

[4] It also illustrates how costly is for firms to discover their offshoring potential in every possible location.

sults as predicted by the models with perfect information, particularly when those changes are not able to sufficiently affect firms' beliefs[5]. In consequence, this paper may bring new insights about the effectiveness of institutional reforms or ambicious public investment programs in infrastructure implemented by governments with the aim of promoting the insertion of domestic firms in global production networks.

I present a review of the literature in section 1.1, including the small but growing one on uncertainty in global sourcing. In section 2.2, I introduce the baseline model for two countries, and I extend this to a multi-country model in section 3. Finally, in section 4, I summarize the main conclusions and I briefly describe further possible extensions.

1.1 Literature review

The model is mainly related to the literature on global sourcing with heterogeneous firms, in particular to those developed by Antràs and Helpman (2004, 2008). However, I consider instead a complete contracts environment[6], and I introduce uncertainty in the per period organisational fixed costs abroad. To my knowledge, this paper is the first in the global sourcing literature to introduce uncertainty in the form of a diffuse knowledge about conditions abroad. In addition, it is also the first within this literature that allows firms to learn about their offshoring potential by exploiting informational externalities derived from the behaviour of other firms, and that analyses the welfare implications of this kind of uncertainty in the context of sourcing models.

In regard to the literature on uncertainty in global sourcing, the attention has centered mainly on how the exposure to shocks affects firms' choices. Carballo (2016) examines how the various organisational types of global sourcing respond differently to demand shocks. Kohler et al. (2018), on the other hand, develops a theoretical model to explain the sourcing decisions of firms when they face shocks in demand (the size of the market) or in supply (supplier's productivity) conditions, and analyses the role of labour market institutions (rigidity vs. flexibility) in the firms' choices.

While in the literature mentioned above firms optimise their sourcing strategy under a perfect knowledge about the distributions of shocks -i.e. about the stochastic nature of the world-, in this model firms face an imperfect knowledge about the institutional fundamentals abroad, which are not inherently stochastic, and they are able to progressively reduce their prior uncertainty by exploiting informational externalities.

There is a comparatively more extensive literature on uncertainty in export

[5] I show that when the economy converges to a ”bad” steady state, the country which has better fundamentals but does not receive any offshoring flows must concentrate the reforms in changing perceptions (prior uncertainty), instead of improving fundamentals. See section 3 for a deep analysis.

[6] In upcoming extensions of the model, I consider both cases, incomplete contracts and contractual frictions.

models. As in the previous case, most of it centers the attention on the study of the export behaviour of firms when they are exposed to shocks. As in the previous cases, they usually have a perfect knowledge about the stochastic nature of the world (Rob and Vettas, 2003; Ramondo et al., 2013; Handley, 2014). However, some scholars in this literature have studied export decisions under uncertainty in contexts where firms may improve their prior knowledge by learning, and thus better assess their exporting potential (Segura-Cayuela and Vilarrubia, 2008; Albornoz et al., 2012; Nguyen, 2012; Aeberhardt et al., 2014; Araujo et al., 2016). Among the cited literature, Segura-Cayuela and Vilarrubia (2008), Albornoz et al. (2012) and Araujo et al. (2016) are the closest to my approach, although applied to a different context.

As mentioned above, this paper also complements the literature on institutions and (endogenous) comparative advantages. In this regard, Acemoglu et al. (2007) and Costinot (2009) analise how institutions shape the countries' specialisation profile. In the first case, the analysis concentrates in the technology choices of firms, and thus the sectoral specialization driven by those choices. Instead, Costinot (2009), under a transaction costs approach, characterises the firms' choices about the complexity of the production processes, which depend on the institutional quality prevailing in each country. A common feature of both models is that institutional fundamentals are the underlying factors defining the comparative advantages and thence the sectoral specialisation of the countries.

The multi-country model I introduce here complements these approaches, in particular by showing the role that uncertainty plays in offshoring decisions. While in Acemoglu et al. (2007) and Costinot (2009) the differences in the fundamentals of contractual institutions are the source of comparative advantages, in this model the comparative advantages are a result of the informational spillovers (due to selection patterns) and the institutional fundamentals. In other words, not only the differences in institutional fundamentals matters. The beliefs of the firms and the resulting informational spillovers are a source of comparative advantages, in the sense that they are key factors driving the offshoring flows and the consequential specialisation of countries.

However, the intention of this paper is not aimed at neglecting the importance of the fundamentals, but to point out the limitations of focusing exclusively in them without accounting for the effects of uncertainty and the informational externalities. The characterisation of the multiple equilibria of the multi-country model remarks the importance of both dimensions (beliefs and fundamentals) in defining countries' comparative advantages.

To conclude, the dynamic model is characterised as a Markov decision process, in which firms learn by a Bayesian recursive learning mechanism. In this regard, the closest literature to my approach refers to the models of Rob (1991) and Segura-Cayuela and Vilarrubia (2008), and to the general literature on recursive methods and statistical decisions such as Stokey and Lucas (1989); DeGroot (2005); Sutton and Barto (2018). Rob (1991) introduces a model of market entry where there is imperfect information about the demand conditions, in particular the size of the market. Rob introduces a Bayesian learning process, which allows firms to progressively improve the information about the demand conditions, characterizing a sequential entry into the market. Based on Rob (1991), Segura-Cayuela and Vilarrubia (2008) applies this same approach to a Melitz (2003)'s type model with uncertainty in fixed exporting costs, leading to sequential entry in the foreign market.

2 The two-country model: North-South

The model consists of a world economy with two countries, North (N) and South (S), and a unique factor of production, labour (ℓ). I assume Complete Contracts (CC).

Preferences. They are represented by a per period Cobb-Douglas utility function

$$U = \gamma_0 \ln q_0 + \sum_{j=1}^{J} \gamma_j \ln Q_j \quad , \quad \sum_{j=0}^{J} \gamma_j = 1 \tag{1}$$

where q_0 is the per period consumption of a homogeneous good, and Q_j is an index of the per period aggregate consumption in the differentiated sectors $j = \{1, ..., J\}$.

All the goods are tradable in the world market, and there are no transport costs nor trade barriers, i.e. there is free trade in the final good markets and in intermediate inputs. Also, I assume consumers have identical preferences across countries.

The per-period aggregate consumption in a differentiated sector j is a constant elasticity of substitution (CES) function denoted by

$$Q_j = \left[\int_{i \in I_j} q(i)^{\alpha_j} di \right]^{1/\alpha_j} \quad , \quad 0 < \alpha_j < 1 \tag{2}$$

which consists on the aggregation of the variety consumption $q_j(i)$ on the range of varieties i in sector j. The elasticity of substitution between any two varieties in this sector is $\sigma_j = 1/(1 - \alpha_j)$.

The inverse demand function for variety i in differentiated sector j:

$$p_j(i) = \gamma_j E Q^{-\alpha_j} q_j(i)^{\alpha_j - 1} \tag{3}$$

where E denotes the per period total (world) expenditure, and the price index in each differentiated sector j is defined as:

$$P_j \equiv \left[\int_{i \in I} p_j(i)^{1-\sigma_j} di \right]^{\frac{1}{1-\sigma_j}} \tag{4}$$

Technology and production in differentiated sectors ($j \in \{1, ...J\}$). The per period output of variety i is produced with a Cobb-Douglas technology:

$$q_j(i) = \theta \left(\frac{x_{h,j}(i)}{\eta_j} \right)^{\eta_j} \left(\frac{x_{m,j}(i)}{1-\eta_j} \right)^{1-\eta_j} \tag{5}$$

where the respective inputs are the headquarter services, $x_{h,j}$, and the intermediate input, $x_{m,j}$. They are respectively supplied by the headquarter[7], H, and the intermediate input supplier, M.

$\eta_j \in (0, 1)$ is a technology parameter, which measures the headquarter-services intensity of the sector, and the parameter θ represents the firm's productivity level, which varies across firms.

Both inputs for each variety are produced with constant return technologies:

$$x_{k,j}(i) = \ell_{k,j}(i) \quad \text{with } k = h, m \tag{6}$$

Assumption A. 1. *The headquarter services and the final-good varieties can be produced only by firms in the North.*

Intuitively, this implies that northern firms are the only ones who count with the knowledge and capacity to supply the services $x_{h,j}$. Therefore, the final-good producers in the differentiated sectors are always located in North (Antràs and Helpman, 2004).

Entry cost and productivity draw. The process corresponds to a Melitz (2003)'s entry mechanism. Firms must pay a one-period market entry sunk cost $f_{e,j}$ in northern units of labour, i.e. $w^N f_{e,j}$. After the payment, they discover their productivity θ, which is drawn from a c.d.f. denoted by $G(\theta)$. This entry cost can be thought as the R&D expenditures that the firm has to afford in order to develop the variety she will commercialize.

Technology in homogenous sector ($j = 0$). I assume that the homogenous sector has a constant returns to scale technology:

$$q_0 = A_{0,l}\ell_0 \tag{7}$$

where $A_{0,l} > 0$ is a productivity parameter in country l.

Assumption A. 2. *The productivity of northern workers in the homogeneous good sector is higher than southern workers in the same sector, i.e.* $A_{0,S} < A_{0,N}$. *Therefore,* $w^N > w^S$.

Furthermore, I assume that γ_0 is large enough such that the homogeneous good is produced in every country[8].

[7] I refer to the final-good producer alternatively as the firm, the headquarter, HQ, H.

[8] This assumption implies that wages remain constant. In an upcoming version of the paper, I relax this assumption allowing wages to respond to offshoring flows.

2.1 Perfect information equilibrium

The equilibrium characterised in this section plays an important role in the study of the transition path and the convergence properties of the dynamic model with uncertainty. In the remaining sections of the paper, the analysis focuses in the sectoral dynamics of one differentiated sector j. Therefore, for simplicity, I assume there is only one differentiated sector, and thus drop the subscript j.

The equilibrium with perfect information described here is closely related to the Antràs and Helpman (2004)'s model with two main differences. First, in this case I present a model with complete contracts, instead of incomplete contracts[9]. Second, I introduce a market research sunk cost f^r in northern labour units, which must be paid in advance by those firms who want to offshore[10]. This offshoring sunk cost can be understood as the market research costs and feasibility studies that a firm has to afford when analyses potential suppliers in different locations.

The figure 2.1 shows a schematic representation of the timing of events under perfect information. For a detailed solution of the model under perfect information see Appendix A.

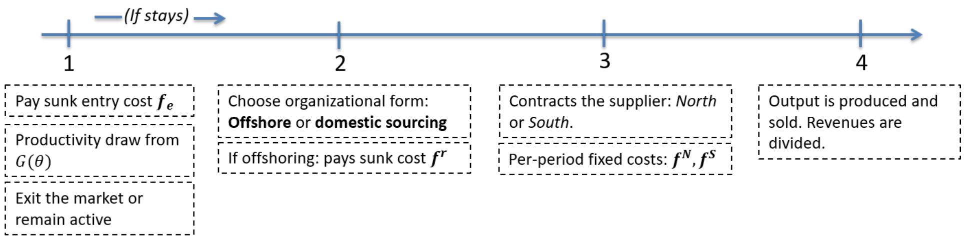

Figure 1: Timing of events.

Assumption A. 3. *The ranking of per-period fixed production costs is* $f^N < f^S$

For the ordering of organisational types analysed in this paper, and in order to avoid a full taxonomy of cases, I assume:

Assumption A. 4. *The relation between fixed costs and wages is given by:*

$$\frac{f^S + (1-\lambda)f^r}{f^N} > \left(\frac{w^N}{w^S}\right)^{(1-\eta)(\sigma-1)}$$

with $\lambda \in (0, 1)$ denoting the per-period survival rate to an exogenous "death" shock that pushes the firm out of business.

[9]This reduces the sourcing decision only to the location dimension. Instead, under incomplete contracts and a property rights approach to the theory of the firm, the sourcing choice involves the location and the ownership dimensions.

[10]The offshoring sunk cost f^r does not play an important role in the model with perfect information, but as I will show in section 2.2, it makes firms costly (and risky) to explore their offshoring potential under uncertainty.

Under complete contracts, the organisational choice reduces to the location dimension, i.e. the firm must choose whether she will source domestically or from a foreign location. Thence, it is possible to define the *per-period offshoring profit premium* of a firm θ as:

$$\begin{aligned}\pi^{S,prem}(\theta) &\equiv \pi^{S}(\theta) - \pi^{N}(\theta) \\ &= \frac{r^{N}(\theta)}{\sigma}\left[\left(\frac{w^{N}}{w^{S}}\right)^{(1-\eta)(\sigma-1)} - 1\right] - w^{N}\left[f^{S} - f^{N}\right]\end{aligned} \tag{8}$$

A final good producer chooses the organisational form that maximizes their lifetime profits. Under perfect information, this is equivalent to choose the sourcing strategy that maximizes the per-period profits. Hence, using equation (8), firms prefer to offshore whenever the per-period offshoring profit premium is higher than (or equal to) the discounted offshoring sunk cost. Formally,

$$\pi^{S,prem}(\theta)\begin{cases} < (1-\lambda)w^{N}f^{r} & \text{if } \theta < \theta^{S,*} \Rightarrow \text{ firm } \theta \text{ sources domestically} \\ = (1-\lambda)w^{N}f^{r} & \text{if } \theta = \theta^{S,*} \Rightarrow \text{ firm } \theta \text{ offshore} \\ > (1-\lambda)w^{N}f^{r} & \text{if } \theta > \theta^{S,*} \Rightarrow \text{ firm } \theta \text{ offshore} \end{cases} \tag{9}$$

with $\theta^{S,*}$ indicating the offshoring productivity cutoff [11]. The $*$ refers to the equilibrium values of the variables with perfect information.

Figure 2 illustrates the offshoring productivity cutoff ($\theta^{S,*}$) and the market entry productivity cutoff ($\underline{\theta}^{*}$) at equilibrium. The dark area in between the profit curves represent the per-period offshoring profit premium for each firm θ with a productivity above the offshoring cutoff.

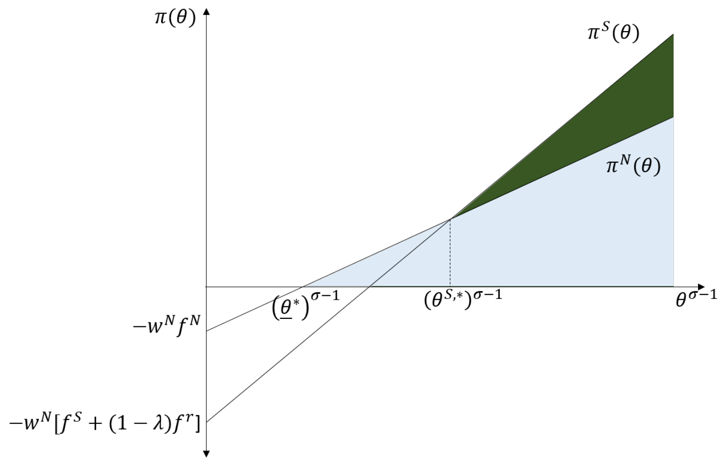

Figure 2: Per-period offshoring profit premium.

[11]The Antràs and Helpman (2004)'s model with complete contracts and no offshoring sunk cost would be represented as the case where the marginal offshoring firm earns $\pi^{S,prem}(\theta^{S,*}) = 0$, i.e. the offshoring cutoff firm obtains zero offshoring profit premium at equilibrium.

2.2 The North-South global sourcing dynamic model with uncertainty

This section focuses in the study of the sourcing decisions when firms face uncertainty with respect to the per-period fixed costs of production in the South. However, firms can exploit informational spillovers, and thence they can update their knowledge and progresively reduce their prior uncertainty.

The dynamic approach of this model is important for the emergence of the informational externalities that are exploited by the firms in their learning mechanism. Firms are able to observe other firms' behaviour and thus update their prior knowledge about institutional conditions in the South.

In the following section I introduce the timing of events of the dynamic model with uncertainty, and in section 2.2.3 I describe the emergence of the informational externalities and the configuration of the learning mechanism.

I define the initial conditions of the dynamic model as the steady state of an economy with non-tradable intermediate inputs ($n.t.i.$), i.e. a situation where the final-good producers can source only domestically but the final goods are tradable in the world market. This may reflect a situation where pre-existing (beliefs about) institutions in the South make the cost of offshoring prohibitively high[12].

At $t = 0$ there is an unexpected shock in which intermediate inputs become tradable, i.e. offshoring becomes possible at least initially for some firms (the more productive ones). Continuing the example from above, at $t = 0$ the southern government annouces that it has finished the implementation of a deep institutional reform with the aim of attracting northern firms to hire southern intermediate inputs suppliers. Nevertheless, northern firms do not fully believe in the announcement of the foreign government, but they know that some changes have been implemented. Therefore, northern firms form a prior belief about the possible scope of these reforms, which may turn offshoring attractive to some of the firms. These prior beliefs manifest as a prior distribution about the southern institutions[13].

Under perfect information, after the announcement the adjustment to the full information equilibrium characterised in the previous section is instantaneous. However, in the following sections I show that under uncertainty the adjustment is sequential and it is led by the most productive firms in the market.

2.2.1 Timing of events

Figure 3 illustrates the timing of events after the intermediate inputs market opens up to trade. From $t = 0$ on, firms sourcing domestically can choose whether to explore their offshoring potential or wait for new incoming information. If the firm chooses offshoring, then she pays the offshoring sunk cost f^r and discovers the true fixed cost f^S (this fixed cost remains as private information of the firm). Therefore,

[12]Or a case where the beliefs about the quality of the infrastructure in the foreign country is too low in order to make offshoring attractive for northern firms.

[13]Regarding the infrastructure example, firms may not believe completely in the scope and quality of the investments program in infrastructure, but they know that some investments have been executed. Therefore, they build a prior belief about the new prevailing conditions in South.

she can take the optimal sourcing decision with complete certainty for the rest of the periods. In terms of the Markov decision process, choosing to explore the offshoring potential by paying the sunk cost f^r is an absorbing state for a firm θ.

However, if the firm decides to wait for more information to be revealed, then, while she is waiting, she keeps sourcing domestically with a northern supplier. On the following period she must decide again whether to explore her offshoring potential or wait, but now under a reduced uncertainty given the new information revealed by observing the behaviour of the new offshoring firms, i.e. those firms that have explored offshoring in the previous period.

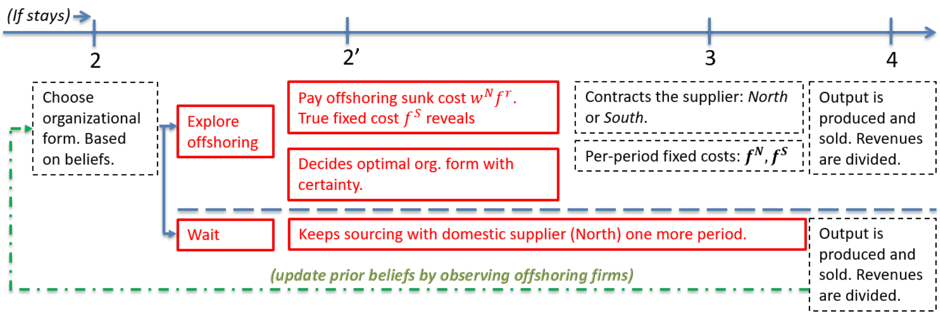

Figure 3: Timing of events - Uncertainty.

Assumption A. 5. *Firms are risk neutral*

2.2.2 Initial conditions: non-tradable intermediate inputs (n.t.i.)

I characterise briefly the steady state of the n.t.i. economy, which defines the initial conditions for the dynamic model. For a detailed solution see Appendix B.

Given Assumption A.2, the prices charged by a firm θ under domestic sourcing are higher than under offshoring. Formally,

$$p(\theta) = \frac{w^N}{\alpha\theta} > \frac{(w^N)^\eta (w^S)^{1-\eta}}{\alpha\theta} = p^{\text{off}}(\theta)$$

Therefore, it is straightforward to see that[14]

$$P^{n.t.i.} > P^* \quad ; \quad Q^{n.t.i.} < Q^* \quad ; \quad \underline{\theta}^{n.t.i.} < \underline{\theta}^*$$

where superscript $n.t.i.$ indicates the equilibrium value for the non-tradable intermediate inputs economy, and $*$ still refers to the equilibrium variables under perfect information with tradable intermediate inputs.

The higher initial price index allows less productive firms to remain active in the market after entry, which is represented by a lower market productivity cutoff.

[14]Regarding the expressions of the Tradable Intermediate Inputs economy with perfect information, see Appendix C.

However, when offshoring becomes possible for northern firms, the least productive firms in the market are not able to face the stronger competition which pushes down the price index, and therefore they must leave the market. This expulsion from the market of the least productive firms increases *pari passu* the offshoring productivity cutoff reduces.

Additionally, it is possible to observe a polarisation effect as in Melitz (2003), but of a different nature. The polarisation effect comes instead from the cost advantages that firms doing offshoring can exploit by obtaining access to foreign suppliers with lower marginal costs for the production of the intermediate inputs.

Welfare implications. The comparison between the $n.t.i.$ and the $*$ scenarios above clearly shows the welfare gains from offshoring. This derives from the fact that, in the steady state, the economy $*$ reaches a lower price index and thus a higher agregate consumption in the differentiated sectors, and those gains are larger the higher is the share of offshoring firms.

In regard to this last result, the price index of the $*$ economy is given by:

$$(P^{*})^{1-\sigma} = \left(P^{\text{n.t.i.}}\right)^{1-\sigma} + \chi^{*}\left[\left(\frac{w^{N}}{w^{S}}\right)^{(1-\eta)(\sigma-1)} - 1\right]\left(P^{\text{off}|\text{n.t.i.}}\right)^{1-\sigma} \tag{10}$$

where χ^{*} denotes the share of offshoring firms in the steady state of the economy, and $P^{\text{off}|\text{n.t.i.}}$ refers to the price index of the offshoring firms if they would be sourcing domestically[15]. Therefore, it is easy to observe that the price index P^{*} is decreasing in χ^{*}.

2.2.3 Dynamic model with uncertainty: tradable intermediate inputs

In the remaining of this section, I present the dynamic model with tradable intermediate inputs, and introduce uncertainty in the per period organisational fixed cost in South[16].

The dynamic model is characterised as a Markov decision process in which firms learn by exploiting the informational externalities that emerge from other firms' behaviour. In particular, I assume that firms are able to observe the total revenues of the market, the market share of every active firm, and the type of sourcing strategy chosen by each of her competitors. Using this information together the known wages at each location, every firm can infer with exact precision the productivity level of each of her competitors.

The state of the Markov process is characterised by a state with two dimensions: "beliefs" and "physical". The second corresponds to the data observed by the firms in the market, i.e. the per-period informational externalities produced by

[15] $P^{\text{n.t.i.}}$ requires a careful interpretation in the context of the $*$ economy. As it is possible to observe in Appendix C, it corresponds to the n.t.i. price index but considering the cutoff productivity of the $*$ economy, i.e. $\underline{\theta}^{*}$. Nevertheless, when $\chi^{*} \to 0$, the market cutoff $\underline{\theta}^{*} \to \underline{\theta}^{n.t.i.}$ and $P^{*} \to P^{n.t.i.}$.

[16] For a solution of the tradable intermediate inputs economy under perfect information, and its comparison with the n.t.i. economy, see Appendix C.

offshoring firms. On the other hand, the "beliefs" dimension refers to the Bayesian learning mechanism by which firms update their knowledge and reduce their prior uncertainty, exploiting the new incoming data. Below, I describe both state dimensions in more detail.

"Beliefs" state: Prior uncertainty and Bayesian learning. At $t = 0$, the institutional reform in the southern country allows northern firms to consider offshoring as a potentially feasible sourcing strategy. Nevertheless, as mentioned above, they do not fully believe in the announcement of the southern government. Thus, northern firms form a prior (diffuse) knowledge about the scope of the reforms and the quality of the institutions abroad. Formally, this prior uncertainty translates as a prior distribution of the per-period organisational fixed costs in South represented by:

$$f^S \sim Y(f^S) \quad \text{with} \quad f^S \in [\underline{f}^S, \bar{f}^S]$$

where $Y(.)$ denotes the c.d.f. of the prior distribution.

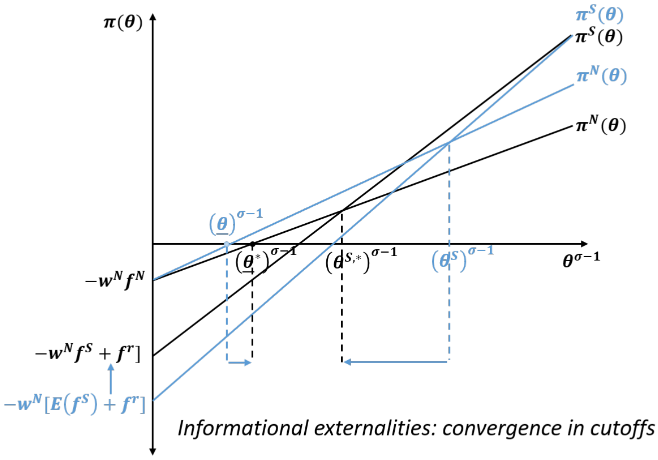

Figure 4: Perfect information and static equilibrium with uncertainty.

Figure 4 illustrates the perfect information equilibrium (*dark lines*) compared to the expected profits by organisational type given the initial prior uncertainty (*light lines*). The latter would represent the equilibrium of a static model with uncertainty, in which firms cannot learn through informational spillovers. However, as it is shown below in this model, the dynamic approach captures the emergence of informational externalities and characterises the conditions under which the steady state of the economy converges progressively to the perfect information equilibrium. In this regard, informational externalities play a key role by allowing firms to progressively discover their offshoring potential and thence adjust their sourcing strategy.

The learning mechanism takes the form of a recursive Bayesian learning process, in which the posterior distribution for any $t > 0$ is given by:

$$f^S \sim \begin{cases} Y(f^S | f^S \leq f_t^S) = \frac{Y(f^S | f^S \leq f_{t-1}^S)}{Y(f_t^S | f^S \leq f_{t-1}^S)} & \text{if } \tilde{f}_t^S = f_t^S < f_{t-1}^S \\ f_t^S & \text{if } \tilde{f}_t^S < f_t^S \end{cases}$$

with f_t^S defined as the revealed upper bound in t, and $\tilde{f}_t^S$ as the expected revealed upper bound in t. Both are defined more precisely below in the "physical" state, as they are both related to the data -informational externalities- that firms receive every period.

Assumption A. 6. *The prior distribution satisfies the following condition:*

$$\frac{\partial [f_t^S - E(f^S | f^S \leq f_t^S)]}{\partial f_t^S} > 0$$

Intuitively, this assumption implies that information flows are decreasing as the upper bound of the distribution reduces.

"Physical" state. Let's define $f^S(\theta)$ as the maximum affordable fixed cost in South for a firm with productivity θ. This implies that the firm θ earns zero per period offshoring profit premium if this is the per-period fixed costs in South. Formally:

$$\pi^{S,prem}(\theta) = 0 \Rightarrow f^S(\theta) = \frac{r^{N,*}(\theta, Q_t)}{\sigma w^N} \left[\left(\frac{w^N}{w^S} \right)^{(1-\eta)(\sigma-1)} - 1 \right] + f^N$$

I define θ_t as the least productive firm doing offshoring in period t. This implies that after paying the offshoring sunk cost $w^N f^r$, the firm θ_t remains sourcing from South if she receives non-negative per-period offshoring profit premium, i.e. $\pi^{S,prem} \geq 0$. Therefore, f_t^S, i.e. the revealed upper bound in t, represents the maximum affordable fixed cost in South such that firm θ_t remains sourcing from abroad after paying the offshoring sunk cost in $t-1$, and it is given by:

$$f_t^S \equiv f^S(\theta_t) = \frac{r^N(\theta_t, Q_t)}{\sigma w^N} \left[\left(\frac{w^N}{w^S} \right)^{(1-\eta)(\sigma-1)} - 1 \right] + f^N \tag{11}$$

Finally, let's define $\tilde{\theta}_t$ as the least productive firm trying offshoring in $t-1$. Therefore, $\tilde{f}_t^S \equiv f^S(\tilde{\theta}_t)$, i.e. the expected revealed upper bound in t, represents the maximum affordable fixed cost in South such that this firm would remain offshoring after paying the sunk cost in $t-1$.

Both, $\tilde{\theta}_t$ and θ_t are observable by the firms that are waiting and sourcing domestically. Thus, $\tilde{f}_t^S$ and f_t^S can be easily computed by them. These two variables, $\tilde{f}_t^S$ and f_t^S, are key elements defining the incoming data of the Bayesian learning mechanism in the "beliefs" dimension of the Markov state.

When both values coincide, the true fixed cost has not been revealed, but firms can truncate their prior distribution according to the Bayesian rule explained above.

However, when they differ, i.e. $\tilde{f}_t^S < f_t^S$, those firms in the range $\theta \in [\tilde{\theta}_t; \theta_t)$ have explored their offshoring potential, and after discovering the true organisational fixed cost in South, f^S, they have decided to remain sourcing domestically. This situation reveals that the marginal offshoring firm which earns zero offshoring profit premium is θ_t and thence the true value $f^S = f_t^S$. After this event, the learning process stops.

Offshoring decision. At any period t, a firm sourcing domestically, who has never explored offshoring, must decide whether to discover her offshoring potential by paying the sunk cost, or wait for new information to be released. The option of waiting is a direct result of the existence of informational externalities, through which firms can reduce the risk of the offshoring decision by waiting for new information and truncate their prior uncertainty.

Formally, the firm solves the value function $\mathcal{V}_t(\theta; \theta_t)$:

$$\mathcal{V}_t(\theta; \theta_t) = \max \{V_t^o(\theta; \theta_t); V_t^w(\theta; \theta_t)\}$$

with $V_t^o(\theta; .)$ denoting the value of offshoring and $V_t^w(\theta; .)$ denoting the value of waiting for a firm with productivity θ in t.

The existence of informational externalities has two important consequences in the outcomes of the model. First, some firms with a positive expected offshoring profit premium may decide to delay the offshoring exploration, in order to reduce the risk of the decision. Second, some firms that have initially a negative expected offshoring profit premium may find out, after enough information has been released, that it may be profitable for them to explore offshoring. This trade-off "explore or wait" is a key element of the characterisation of firms' decisions.

The value of offshoring in period t is given by the discounted expected total offshoring profit premium that the firm can earn starting from t minus the sunk cost f^r, or a loss equivalent to the sunk cost in the case that after paying it she finds out that the offshoring premium is negative. Formally, the value of offshoring in t for a firm θ is given by:

$$V_t^o(\theta; \theta_t) = \mathbb{E}_t \left[\max \left\{ 0; \sum_{\tau=t}^{\infty} \lambda^{\tau-t} \pi_\tau^{S,prem}(\theta) \right\} \middle| f^S \leq f_t^S \right] - w^N f^r$$

One important feature of the exploration of the offshoring potential is that after paying the offshoring sunk cost, the firm has no remaining uncertainty about the fixed costs. Thence, she chooses her optimal sourcing strategy for all remaining periods. In other words, exploring offshoring leads those firms to an absorbing state of the Markov decision process.

The value of waiting at period t for a firm θ is given by:

$$V_t^w(\theta; \theta_t) = 0 + \lambda \mathbb{E}_t \left[\mathcal{V}_{t+1}(\theta; \theta_{t+1}) \right]$$

The first term of RHS means that the firm remains doing domestic sourcing in t, and therefore earns zero offshoring profit premium in t. The second term implies that she decides again in the following period whether to explore her offshoring potential or wait.

The Bellman's equation can be expressed as:

$$\mathcal{V}_t(\theta;\theta_t) = \max\left\{V_t^o(\theta;\theta_t); \lambda\mathbb{E}_t\left[\mathcal{V}_{t+1}(\theta;\theta_{t+1})\right]\right\}$$

By *Assumption A.6*, given the information set in t, waiting for one period and explore offshoring in the following one, $V_t^{w,1}(.)$, dominates waiting for longer periods.

$$V_t^{w,1}(\theta;\theta_t,\theta_{t+1}) > V_t^{w,2}(\theta;\theta_t,\theta_{t+2}) > .... > V_t^{w,n}(\theta;\theta_t,\theta_{t+n})$$

Therefore, the One-Step-Look-Ahead (OSLA) rule is the optimal policy (see Appendix D), and the Bellman's equation becomes:

$$\mathcal{V}_t(\theta;.) = \max\left\{\mathbb{E}_t\left[\max\left\{0;\sum_{\tau=t}^{\infty}\lambda^{\tau-t}\pi_\tau^{S,prem}(\theta)\right\}\middle| f^S \le f_t^S\right] - w^N f^r; V_t^{w,1}(\theta;.)\right\} \tag{12}$$

A *trade-off function* can be obtained by a further transformation of equation (12). This function determines the offshoring decision at any period t for any firm θ:

$$\mathcal{D}_t(\theta;\theta_t,\tilde{\theta}_{t+1}) = V_t^o(\theta;\theta_t,\tilde{\theta}_{t+1}) - V_t^{w,1}(\theta;\theta_t,\tilde{\theta}_{t+1}) \tag{13}$$

where the first argument of $\mathcal{D}_t(.)$ indicates the productivity of the firm taking the decision, the second argument refers to the state of the system at t, i.e. the productivity of the least productive offshoring firm in South, and the third argument denotes the expected new information that will be revealed at t, i.e. the least productive firm that will attemp offshoring in t.

At any time t, firm's offshoring decision is based on:

$$\mathcal{D}_t(\theta;.)\begin{cases}\ge 0 \Rightarrow & \text{pay the sunk cost and discover the offshoring potential.}\\ < 0 \Rightarrow & \text{remain sourcing domestically for one more period.}\end{cases}$$

By plugging the respective expressions of $V_t^o(\theta;.)$ and $V_t^{w,1}(\theta;.)$ in the trade-off function, it is possible to derive Proposition 1 (see proofs in Appendix D).

Proposition 1 (Sequential entry)**.** *Firms with higher productivity have an incentive to explore offshoring in early periods.*

$$\frac{\partial\mathcal{D}_t(\theta;\theta_t,\tilde{\theta}_{t+1})}{\partial\theta} \ge 0$$

Another way to express the result coming from Proposition 1 is: *firms explore offshoring sequentially, led by the most productive ones in the market.*

Using Proposition 1, the trade-off function becomes (see proof in Appendix D):

$$\mathcal{D}_t(\theta;\theta_t,\tilde{\theta}_{t+1}) = \max\left\{0;\mathbb{E}_t\left[\pi_t^{S,prem}(\theta)\middle| f^S \le f_t^S\right]\right\} - w^N f^r\left[1 - \lambda\frac{Y(f_{t+1}^S)}{Y(f_t^S)}\right] \tag{14}$$

with $\frac{Y(f_{t+1}^S)}{Y(f_t^S)} \equiv Y(f_{t+1}^S|f^S \le f_t^S)$.

Assumption A. 7. *At least the most productive firm in the market finds profitable to offshore, given the prior knowledge at* $t = 0$.

$$\mathcal{D}_{t=0}(\bar{\bar{\theta}}; \bar{\bar{\theta}}, \bar{\bar{\theta}}) > 0$$

where $\bar{\bar{\theta}}$ *refers to the most productive firm in the market.*

Intuitively, this means that at least the most productive firm must find profitable to explore offshoring given the prior knowledge. This assumption is key in order to trigger the production of informational externalities and lead to the equilibrium path defined by the trade off function[17].

Proposition 2 (Per-period offshoring cutoff)**.** *The offshoring productivity cutoff at any period t,* $\tilde{\theta}_{t+1}$*, is defined as the fixed point in the trade-off function*

$$\begin{aligned}\mathcal{D}_t(\tilde{\theta}_{t+1}; \theta_t, \tilde{\theta}_{t+1}) =& 0\\ \mathbb{E}_t\left[\pi_t^{S,prem}(\tilde{\theta}_{t+1})\middle| f^S \leq f_t^S\right] =& w^N f^r\left[1 - \lambda\frac{Y(f_{t+1}^S)}{Y(f_t^S)}\right]\end{aligned}$$

Thus, solving for $\tilde{\theta}_{t+1} \equiv \theta_t^S$, the offshoring cutoff at period t is

$$\theta_t^S = [(1-\gamma_0)E]^{\frac{\sigma}{1-\sigma}}\,\tilde{Q}_{t+1}\left[\frac{w^N\left[\mathbb{E}_t(f^S|f^S \leq f_t^S) - f^N + \left(1 - \lambda\frac{Y(f_{t+1}^S)}{Y(f_t^S)}\right)f^r\right]}{\psi^S - \psi^N}\right]^{\frac{1}{\sigma-1}}$$

with $\tilde{Q}_{t+1}$ denoting the aggregate consumption under an offshoring productivity cutoff defined by $\tilde{\theta}_{t+1}$, i.e. $\tilde{Q}_{t+1} \equiv Q(\tilde{\theta}_{t+1})$.

Long-run properties (convergence analysis). The remaining part of this section concentrates in the characterisation of the steady state of the economy, and the conditions under which it converges to the perfect information steady state defined in section 2.1.

Formally, it is equivalent to show that convergence to perfect information steady state holds by any of the following conditions:

$$f_t^S \xrightarrow{t\to\infty} f^S \quad ; \quad \theta_t^S \xrightarrow{t\to\infty} \theta^{S,*} \quad ; \quad \underline{\theta}_t \xrightarrow{t\to\infty} \underline{\theta}^*$$

In first place, it is possible to observe that in the long-run the learning mechanism collapses in the lower bound of the prior distribution, unless the true fixed cost f^S is revealed and the updating process stops in a finite number of periods. Therefore, these cases can be summarized as:

- If $f^S = \underline{f}^S \Rightarrow$ The distribution collapses in the lower bound of the prior.

[17] When the support of the productivity distribution $G(\theta)$ is $[\theta_{min}, \infty)$, e.g. Pareto distribution, it is enough to assume that the prior distribution $Y(f^S)$ has a finite expected value.

- If $f^S \in (\underline{f}^S, \bar{f}^S] \Rightarrow$ Updating stops sooner (true value revealed).

Finally, the last part consists in analysing the behaviour of the trade-off function in the long-run. This implies finding the fixed point as $t \to \infty$, analysing whether this point corresponds to the perfect information steady state, and proving its uniqueness (see proof in Appendix D).

$$\mathcal{D}(\theta_\infty; \theta_\infty, \theta_\infty) = 0$$
$$\mathbb{E}_t \left[\pi^{S,prem}(\theta_\infty^S) \middle| f^S \leq f_\infty^S \right] = w^N f^r \left[1 - \lambda\right]$$

Proposition 3 (Convergence of offshoring productivity cutoff)**.** *The economy converges asymptotically to the full information equilibrium when*

$$\textit{Case I: } f^S = \underline{f}^S \Rightarrow f_\infty^S = \underline{f}^S \Rightarrow \theta_t^S \xrightarrow{t\to\infty} \theta^{S,*}$$

Otherwise, if $f^S > \underline{f}^S$, it leads to "excessive" offshoring converging to

$$\Rightarrow \begin{cases} \textit{Case II: } \theta_t^S \xrightarrow{t<\infty} \theta^{S,\neg r} & \textit{if } \underline{f}^S + (1-\lambda) f^r < f^S \\ \textit{Case III: } \theta_t^S \xrightarrow{t\to\infty} \theta^{S,\neg r} & \textit{if } \underline{f}^S + (1-\lambda) f^r = f^S \\ \textit{Case IV: } \theta_t^S \xrightarrow{t\to\infty} \theta_\infty^S & \textit{if } \underline{f}^S + (1-\lambda) f^r > f^S > \underline{f}^S \end{cases}$$

with $\theta^{S,*} > \theta_\infty^S > \theta^{S,\neg r}$, and $\theta^{S,\neg r}$ denoting the case where the marginal firms obtain zero per period offshoring profit premium, i.e. firms who cannot recover the offshoring sunk cost.

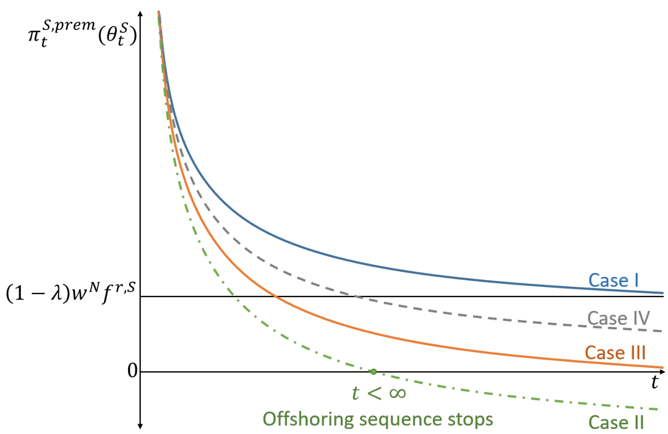

Figure 5: Convergence paths

Proposition 3 shows that there are four possible cases of convergence. The steady state of the economy is unique but to which of the points it converges it depends on distance of the lower bound of the prior distribution with respect to the true value f^S.

Except for the Case I ($f^S = \underline{f}^S$), in which the economy converges to the perfect information steady state in infinite periods, in all the other cases it converges to points where the economy experiences some excesive offshoring. Case II is the only situation in which the true fixed cost in South is revealed in a finite time[18].

Figure 5 illustrates these convergence points. The Case IV corresponds to any point between Case I and III, and the Case II to any point below Case III.

To conclude, as mentioned above, this also leads to the convergence in the market productivity cutoff, pushing the least productive firms out of the market. The increasing number of offshoring firms reduce the sectoral price index, making competition increasingly harder for the least productive firms in the market. Therefore, more firms are pushed out of business until the sequential offshoring dynamic stops.

Proposition 4 (Convergence properties of market productivity cutoff)**.** *Convergence in offshoring productivity cutoff leads to the convergence in sectoral price index and in the aggregate consumption, leading to a convergence in the market productivity cutoff by expelling the least productive firms from the market.*

Therefore, considering the taxonomy of cases from Proposition 3,

$$\begin{cases} \text{Case I: } \underline{\theta}_t \to \underline{\theta}^* & \text{if } \theta_t^S \to \theta^{S,*} \\ \text{Cases II, III: } \underline{\theta}_t \to \underline{\theta}^{\neg r} & \text{if } \theta_t^S \to \theta^{S,\neg r} \\ \text{Case IV: } \underline{\theta}_t \to \underline{\theta}_\infty \in (\underline{\theta}^*; \underline{\theta}^{\neg r}) & \text{if } \theta_t^S \to \theta_\infty^S \in (\theta^{S,\neg r}; \theta^{S,*}) \end{cases}$$

Welfare considerations. I have shown below that the transition from the initial conditions defined by the $n.t.i.$ equilibrium to the $*$ steady state presents potential welfare gains from offshoring. The convergence defined by Propositions 3 and 4 show that in the long run the informational spillovers allow the economy to achieve those welfare gains, as $P_t \downarrow P^*$ and thence $Q_t \uparrow Q^*$.

Summary. In a world with two countries, I observe that when firms face uncertainty about conditions abroad, represented in this case as uncertainty in the per-period organisational fixed cost in South, this affects the optimal sourcing choices compared to the full information equilibrium. In this sense, from an static approach without informational spillovers, it can be easily shown that uncertainty leads to lower profits in the firms who are choosing sub-optimally, while the economy faces higher price-index and a lower aggregate consumption.

From a dynamic perspective, informational externalities arise allowing firms to progressively reduce their prior uncertainty. This results in a sequential offshoring equilibrium path led by the most productive firms in the market. This path converges to (a neighborhood of) the full information steady state of the economy. Therefore, the welfare gains from offshoring are fully realised in the long run.

[18] In Case II, after the true value f^S has been revealed, the ”dead shock effect” will progressively vanish the excessive offshoring.

3 The multi-country model

In a multi-country world, particularly one in which northern firms have alternative foreign locations for their suppliers, two immediate questions arise: i) how is the allocation of suppliers across countries affected by the informational spillovers?; and ii) what are the welfare consequences of uncertainty in the steady state of the economy?

This section gives a step towards answering those questions. I assume a world economy with three countries: North (N), East (E), and South (S). The final-good producers of the differentiated sectors are still located in North, but now they can choose the location of the intermediate input suppliers by selecting among three possible sourcing strategies: domestic sourcing (North), or offshoring in East or in South.

In order to discover their offshoring potential in the South or in the East, firms must pay the country-specific market research sunk cost $f^{r,S}$ or $f^{r,E}$, respectively. Both are expresed in northern labour units. For simplicity, I assume $f^{r,S} = f^{r,E} = f^r$.

Regarding the institutional fundamentals, I assume that they are better in the South that in the East. However, under uncertainty this is unknown to the firms.

Assumption A. 8. *Institutional fundamentals are better in South than in East.*

$$f^S < f^E$$

As before, along this section I characterise the sectoral dynamics of one differentiated sector, therefore I still drop subscript j.

Under perfect information and symmetry in wages, $w^E = w^S$, this assumption implies that firms will offshore only from South[19].

Under uncertainty, as in section 2, the final good producers in the differentiated sector can reduce the risk of their offshoring decision by waiting and learning from other firms' behaviour, i.e. by exploiting informational spillovers. However, given that these externalities are country-specific[20], the behaviour of firms offshoring in one country does not affect firms' beliefs about institutional conditions in the other locations.

3.1 Multi-country model with symmetric wages

In this section I assume symmetry in wages in the offshoring countries.

[19]In an upcoming version of the paper, I develop the extension for asymmetric wages in foreign countries. In this case, South has better institutional fundamentals, but higher wages than East. This leads to a perfect information equilibrium in which the least productive firms in the market source domestically, while the most productive one source from East. The firms with mid-range productivity level choose offshoring from South instead.

[20]In the case where the informational spillovers are also sector-specific, this implies that the dynamics of each differentiated sector are separable. This may potentially imply that spillovers lead to sectoral specialisation of countries.

Assumption A. 9. *South and East have the same labour productivity in the homogeneous sector.*

$$A_{0,S} = A_{0,E} \Rightarrow w^S = w^E$$

I show below that the model leads to multiple equilibria. The steady state of the world economy, and thus the sectoral specialization of countries, depends on both the institutional fundamentals and the beliefs that firms have about the institutions in those countries. This last feature shows the importance of informational spillovers in defining the countries' comparative advantages.

Moreover, these multiple equilibria have different welfare implications. I will characterise below these equilibria and their respective welfare consequences.

It is worth to clarify that in the rest of this section I refer to convergence to the "perfect information equilibrium" in the following sense: the firms doing offshoring are sourcing only from South, and the offshoring productivity cutoff converges to a steady state as defined in Proposition 3 in section 2.2.3 [21].

I begin with the characterisation of firms' offshoring decision and then I continue with the analysis and typification of the multiple equilibria under different beliefs assumptions: *symmetric* and *asymmetric* priors.

3.1.1 Firms' offshoring decision

At any period t, firms sourcing in North decide whether to explore their offshoring potential or wait. In the case of exploring, they have two options: South or East. Therefore, the decision at t for any firm θ, who has never explored her offshoring potential in the past, takes the following form:

$$\begin{aligned}\mathcal{V}_t(\theta;.) &= \max\left\{V_t^{o,S}(\theta;.); V_t^{o,E}(\theta;.); \lambda\mathbb{E}_t\left[\mathcal{V}_{t+1}(\theta;.)\right]\right\} \\ &= \max\left\{\max\left\{V_t^{o,S}(\theta;); V_t^{o,E}(\theta;.)\right\}; \lambda\mathbb{E}_t\left[\mathcal{V}_{t+1}(\theta;.)\right]\right\}\end{aligned}$$

Assuming $V_t^{o,l}(\theta;.)$ is the solution to the $\max\left\{V_t^{o,S}(\theta;); V_t^{o,E}(\theta;.)\right\}$, with $l = E$ or $l = S$, the firm's decision becomes:

$$\mathcal{V}_t(\theta;.) = \max\left\{V_t^{o,l}(\theta;.); V_t^{w,1,l}(\theta;.)\right\}$$

with $V_t^{o,l}(\theta;.)$ as the value of exploring offshoring in country l in period t for firm θ, and $V_t^{w,1,l}(\theta;.)$ as the value of waiting one period and offshoring in country l in the next period.

Intuitively, this process can be thought as a two-stage decision. In the first stage, firms decide the best offshoring location (in expectation at t) among all the

[21]Considering that some cases of Proposition 3 may lead to excessive offshoring compared to the perfect information equilibrium, this implies an abuse of terminology. Nevertheless, I do this in order to simplify the description of all possible multiple equilibria analysed below. Given this warnings about terminology and notation, for the convergence analysis, I denote with $*$ any of the other cases characterised in Proposition 3.

available foreign locations. In the second stage, firms decide whether to explore offshoring in the chosen location or wait.

3.1.2 Case A: Equilibria with symmetric initial beliefs

I assume that both countries are fully symmetric in beliefs. Therefore, firms randomise their location choice at $t = 0$. Given the assumption of continuum of firms, by the law of large number this leads to a half split of the exploring firms into East and South.

The exploration continues in both countries further periods as long as the symmetry in beliefs remains unbroken, i.e. until the true fixed cost in one of the locations is revealed. Given the assumption A.8, this corresponds to the period until the per-period fixed cost in East discloses. However this event may not take place in a finite time.

Case A-I: *Stable steady state with equally distributed offshore across foreign countries.* Considering the results shown in Proposition 3, the transition path and the steady state of the economy depends on whether the prior beliefs about the eastern institutions are "optimistic" or "pessimistic". I describe both situations below.

Pessimistic beliefs. I define the priors as pesismistic when the lower bound of the distribution is closed enough to the true value f^E. This corresponds to the Cases I, III and IV of Proposition 3, in which the institutional fundamentals in East (f^E) do not reveal in any finite number of periods.[22] Formally, the pessimistic beliefs are defined as:

$$\underline{f} + (1-\lambda)f^r \geq f^E \geq \underline{f}$$

In this case, this condition above also implies that the difference in institutional fundamentals between South and East is relatively small, i.e.

$$0 < f^E - f^S \leq (1-\lambda)f^r$$

In this situation, the offshoring flow continues indefinitely to both countries. Thence, the economy converges to a steady state in which both countries receive offshoring flows, diverging from the optimal sectoral specialization defined by the fundamentals.

From a welfare perspective, the price index and agregate consumption index converge in the long run to the perfect information steady state of the economy. Therefore, welfare gains from offshoring are fully achieved in the long run, but with a very slow and costly transition phase.

$$\theta_t^S \downarrow \theta_\infty^S = \theta^{S,*} \text{ and } \theta_\infty^E < \infty \Rightarrow P_t \downarrow P^* \Rightarrow Q_t \uparrow Q^*$$

Optimistic beliefs. I consider now the situation in which the prior beliefs are relatively optimistic such that the institutional fundamentals in East are revealed in

[22]Symmetry in beliefs implies: $\underline{f}^S = \underline{f}^E = \underline{f}$ and $\bar{f}^S = \bar{f}^E = \bar{f}$ and in the distribution $Y(.)$.

a finite time, i.e. the situation characterised by the Case II of the Proposition 3. Formally, the condition for optimistic beliefs is given by:

$$\underline{f} + (1-\lambda)f^r < f^E$$

I continue now with the characterisation of the transition phase up to the revelation period and after, taking into account the relocation processes from one offshoring location to the other that may take place during this transition phase.

Revelation period of eastern fixed cost. Let's define $\hat{t}$ as the period in which f^E is revealed, and $\theta_{\hat{t}}$ as the productivity level of the marginal firm that remains doing offshoring in East in $\hat{t}$.

From $\hat{t}+1$ on, the offshoring flow concentrates in South following a sequential dynamic as the one characterised in section 2.2.3. Considering this, it is already possible to predict that the industry converges to a steady state in terms of the offshoring productivity cutoff, price index and aggregate consumption which corresponds to the pefect information equilibrium.

$$\theta_t^S \downarrow \theta^{S,*} \Rightarrow P_t \downarrow P^* \Rightarrow Q_t \uparrow Q^*$$

With respect to the specialization of countries, it may be possible that some firms may keep sourcing from East for certain periods. Nevertheless, different types of relocation processes may take place as the share of offshoring firms increases. I analyse them below.

Relocation dynamic of least productive firms offshoring in East. A relocation process of the least productive firms offshoring in East starts unfailingly as soon as the share of offshoring firms keeps increasing after $\hat{t}$.

The following sequential dynamic pushes the price index further down, driving the least productive firms offshoring from East to earn negative offshoring profit premiums if remain sourcing from that country. Thence, starting by the least productive firms in East, firms stop sequentially sourcing from the eastern country and relocate the supply chain to the South.

For any period $t > \hat{t}$, the industry shows $P_t < P_{\hat{t}}$ and $Q_t > Q_{\hat{t}}$. The offshoring productivity cutoff from the East at any period $t > \hat{t}$ is given by [23]:

$$\theta_t^E = [(1-\gamma_0)E]^{\frac{\sigma}{1-\sigma}} Q_t \left[\frac{w^N \left[f^E - f^N \right]}{\psi^E - \psi^N} \right]^{\frac{1}{\sigma-1}} > \theta_{\hat{t}}^E$$

As new firms keep exploring their offshoring potential in South, the "price index effect" pushes up the offshoring productivity cutoff in East.

The convergence of the industry offshoring productivity cutoff, in this case defined as the offshoring productivity cutoff in South θ_∞^S, determines Q_∞ and thus the steady state level of θ_∞^E. In this regard, the industry offshoring productivity

[23] For the expression of $\theta_{\hat{t}}^E$ see Appendix E.

cutoff θ^S_∞ is defined as in section 2.2.3, with the corresponding price index and aggregate consumption steady state levels $P_\infty \equiv P(\theta^S_\infty)$ and $Q_\infty \equiv Q(\theta^S_\infty)$.

Using this results, the expression for the offshoring productivity cutoff in East in the "steady state" of the industry[24] is defined as:

$$\theta^E_\infty = [(1-\gamma_0)E]^{\frac{\sigma}{1-\sigma}} Q_\infty \left[\frac{w^N \left[f^E - f^N\right]}{\psi^E - \psi^N} \right]^{\frac{1}{\sigma-1}}$$

Therefore, considering this relocation decision of the least productive firms, the steady state is (temporarily[25]) characterised by:

$$\theta^E_\infty < \infty \text{ and } \theta^S_t \downarrow \theta^{S,*} \Rightarrow P_t \downarrow P^* \Rightarrow Q_t \uparrow Q^*$$

Relocation decision of most productive firms offshoring in East. When the difference in institutional fundamentals is large enough to compensate the payment of the offshoring sunk cost in South, a second kind of relocation process from East to South may take place.

The firms offshoring from East with productivity $\theta > \theta^E_\infty$ will not be relocated by the mechanism described above. They still find profitable to source from eastern suppliers, i.e. they do not expect to face negative offshoring profit premium. However, these firms may consider to relocate and source from South when the following condition holds:

$$\mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \lambda^{\tau-t} \pi^{S,prem}_\tau(\theta) | f^S \le f^S_t \right] - \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \lambda^{\tau-t} \pi^{E,prem}_\tau(\theta) | f^S \le f^S_t \right] - w^N f^r \ge 0$$

Intuitively, it means that the expected lifetime gains from relocation are large enough to recover the offshoring sunk cost in South. The relocation of the supply chain involves the payment of the sunk cost f^r.

Solving this equation, it leads to:

$$f^E - \mathbb{E}_t \left[f^S | f^S \le f^S_t \right] \ge (1-\lambda) f^r \tag{15}$$

The relocation takes place whenever this condition holds, which is independent of the productivity level of the firms. It means that whenever the expected institutional quality in South is good enough (compared to eastern institutions), the remaining firms sourcing from East will change their suppliers' location to South. A specific feature of the setting of the model is that this relocation is decided by all firms at the same time[26].

[24] This characterisation considers only this relocation of the least productive firms in East. Therefore, it may not represent the true steady state of the industry. Below I incorporate another type of relocation that may arise in the industry, as well as the long run effect of the death shock.

[25] Temporarily in the sense explained in Footnote 24.

[26] In a future extension, I will consider the cases where countries are asymmetric in wages, i.e. $w^S > w^E$, and in fundamentals $f^S < f^E$. This feature will change the dynamic of this last kind of relocation.

In consequence, depending on the magnitude of the difference in institutional fundamentals between East and South, the industry converges to two possible steady states. I define both of them below as *Case A-II* and *CaseA-III*.

<u>*Case A-II:*</u> *Stable steady state without relocation.* This refers to the situation in which differences in institutional fundamentals between South and East are not large enough, i.e.

$$f^E - f^S < (1-\lambda) f^r$$

Thus, the firms already offshoring in East with productivity $\theta > \theta^E_\infty$ will not relocate to South at any period t.

The steady state of the industry is given by:

$$\theta^E_\infty < \infty \text{ and } \theta^S_t \downarrow \theta^{S,*} \Rightarrow P_t \downarrow P^* \Rightarrow Q_t \uparrow Q^*$$

Regarding to the specialization of countries, half of the firms with productivity $\theta \geq \theta^E_\infty$ remain sourcing from East, while the other half of those firms offshore in South. Thus, the economy converges to a suboptimal sectoral specialization compared to the perfect information equilibrium.

Nevertheless, after the institutional fundamentals in East are revealed, in the long run the "death shock effect" pushes the industry to the optimal production allocation. Therefore, the steady state in the long run is finally characterised by:

$$\theta^E_t \to \infty \text{ and } \theta^S_t \downarrow \theta^{S,*} \Rightarrow P_t \downarrow P^* \Rightarrow Q_t \uparrow Q^*$$

<u>*Case A-III:*</u> *Stable steady state with relocation.* Instead, when differences in institutional fundamentals between South and East are large enough, i.e.

$$f^E - f^S \geq (1-\lambda) f^r$$

those firms already offshoring from East with productivity $\theta > \theta^E_\infty$ will relocate to South in the period $t < \infty$ in which the following condition holds:

$$f^E - \mathbb{E}_t \left[f^S | f^S \leq f^S_t \right] = (1-\lambda) f^r$$

Thus, the economy converges to the perfect information equilibrium as defined in section 2.2.3, in which firms only offshore in South and welfare gains from offshoring are realised. The main difference with respect to the *Case A-II* is that in the situation analysed here the optimal specialization is achieved in a finite period of time, while in the other case it realises in the long run (due to the "death shock effect").

$$\theta^E_t \to \infty \text{ and } \theta^S_t \downarrow \theta^{S,*} \Rightarrow P_t \downarrow P^* \Rightarrow Q_t \uparrow Q^*$$

3.1.3 Equilibria with marginally asymmetric initial beliefs

I characterise here the equilibria when the first movers coordinate in the good equilibrium or in the bad equilibrium. In this regard, I introduce asymmetric beliefs about institutions in East and South in order to induce a coordinated movement in favour of one of the countries in the first period.

I describe below all the possible cases. In order to analyse the strength of the path dependence process, I define the conditions under which the coordinated movement of the first explorers to the good or the bad equilibrium leads to a persistent flow into that initially chosen location.

I also define the cases in which the industry equilibrium path pushes the offshoring flow out of the initially chosen location.

Case B: Coordination in the good equilibrium. I start with the case in which the firms' beliefs about the southern institutional quality are better than those about the East. In other words, countries are asymmetric in terms of the prior uncertainty. In this sense, I assume that the asymmetry is defined as a difference in the upper bound of the prior distributions, i.e.

$$\underline{f}^S = \underline{f}^E = \underline{f} \text{ and } \bar{f}^S = \bar{f}^E - \delta; \text{ with } \delta > 0$$
$$\Rightarrow E_{t=0}(f^S|f^S \leq \bar{f}^S) < E_{t=0}(f^E|f^E \leq \bar{f}^E)$$

At $t = 0$, the favourable beliefs about the South induce the most productive firms to coordinate and explore first their offshoring potential in this country. In consequence, informational externalities emerge with respect to the southern country, while no new information about eastern institutions is revealed.

The perception or beliefs about institutions in each country evolves in the following way:

$$f^E \sim Y(f^E) \text{ with } f^E \in [\underline{f}^E, \bar{f}^E]$$
$$f^S \sim \begin{cases} Y(f^S|f^S \leq f_t^S) & \text{if } \tilde{f}_t^S = f_t^S < f_{t-1}^S \\ f_t^S & \text{if } \tilde{f}_t^S < f_t^S \end{cases}$$

The decision at any period t of a firm θ is given by:

$$\mathcal{V}_t(\theta;.) = \max\left\{V_t^{o,S}(\theta;.); V_t^{w,1,S}(\theta;.)\right\}$$

and the respective trade-off function is:

$$\mathcal{D}_t^S(\theta;\theta_t^S,\tilde{\theta}_{t+1}^S) = \max\left\{0;\mathbb{E}_t\left[\pi_t^{S,prem}(\theta)\middle|f^S \leq f_t^S\right]\right\} - w^N f^r\left[1-\lambda\frac{Y(f_{t+1}^S)}{Y(f_t^S)}\right] \tag{16}$$

Due to the effect of informational externalities, the strategy of exploring the offshoring potential in the South increasingly dominates exploring it in the East. Therefore, the sequential offshoring equilibrium path concentrates in South, while East remains producing only the homogeneous good. This leads the industry to the perfect information steady state.

$$\theta_t^E \to \infty \forall t \text{ and } \theta_t^S \downarrow \theta^{S,*} \Rightarrow P_t \downarrow P^* \Rightarrow Q_t \uparrow Q^*$$

with $\theta_t^E \to \infty \forall t$ denoting the fact that no firm offshores in East in any period t.

To conclude, the industrial specialisation of each country is defined according to the perfect information steady state of the economy, and the welfare gains from offshoring are fully realised in the long run[27].

Whether the economy reaches the steady state in a finite or infinite time depends on the conditions defined by Proposition 3.

Case C: Coordination in the bad equilibrium. I assume now that the firms ex ante believe that eastern institutions are better. Formally, this implies that $\delta < 0$.

$$\underline{f}^S = \underline{f}^E = \underline{f} \text{ and } \bar{f}^S = \bar{f}^E - \delta; \text{ with } \delta < 0$$
$$\Rightarrow E_{t=0}(f^S | f^S \leq \bar{f}^S) > E_{t=0}(f^E | f^E \leq \bar{f}^E)$$

The coordination in the bad equilibrium may be stable or unstable depending on the institutional fundamentals in the East, the size of δ and how optimistic are the prior beliefs of the eastern institutions with respect to the fundamentals.

I characterise below all the possible cases.

Case C-I: Stable bad equilibrium path. Differences in how optimistic are the prior beliefs with respect to the fundamentals of eastern institutions push the economy to different transition phases and steady states. Using the definitions of *"pessimistic"* and *"optimistic"* beliefs from above, I show below the two possible paths.

Pessimistic beliefs. As mentioned above, this represents the situation in which the institutional fundamentals are not be revealed in any finite number of periods. Accordingly, the sequential offshoring process continues in the long run and it concentrates only in the eastern country.

In consequence, the offshoring productivity cutoff, $\theta^E_\infty > \theta^{S,*}$, drives the economy to a higher price index P_∞ and lower aggregate consumption index Q_∞ in the steady state.

$$\theta^S_t \to \infty \forall t \text{ and } \theta^E_t \downarrow \theta^E_\infty > \theta^{S,*} \Rightarrow P_t \downarrow P_\infty > P^* \Rightarrow Q_t \uparrow Q_\infty < Q^*$$

In other words, the economy converges to a bad steady state in which the industry ends up in a suboptimal allocation of production across countries[28], and the potential welfare gains from offshoring are not fully realised in the long run.

Optimistic beliefs. This implies that the institutional fundamentals in the East will be revealed in a finite number of periods.

[27] In the very special case when δ is relatively close to zero and the institutional fundamentals in South are extremely bad, such that the true value f^S is high enough to be revealed in $t = 0$ when first explorers go to South, the firms that have failed in South will explore their offshoring potential in East in $t = 1$. Nevertheless, they will all discover that offshoring from East is not profitable for them either, and they will remain sourcing domestically. In this situation, both fixed costs f^S and f^E are revealed in the first two periods, and no firm offshores from East.

[28] i.e. South remains producing only the homogeneous good while all the offshored production of intermediate inputs has been located in East

I define again $\hat{t}$ as the period in which the true value f^E is revealed[29]. Up to $\hat{t}$, the beliefs evolve according to:

$$f^S \sim Y(f^S) \text{ with } f^S \in [\underline{f}^S, \bar{f}^S]$$
$$f^E \sim \begin{cases} Y(f^E | f^E \leq f_t^E) & \text{if } \tilde{f}_t^E = f_t^E < f_{t-1}^E \\ f_t^E & \text{if } \tilde{f}_t^E < f_t^E \end{cases}$$

The decision at any period $t < \hat{t}$ of a firm θ is given by:

$$\mathcal{V}_t(\theta; .) = \max\left\{V_t^{o,E}(\theta; .); V_t^{w,1,E}(\theta; .)\right\}$$

and the respective trade-off function is represented by:

$$\mathcal{D}_t^E(\theta; \theta_t^E, \tilde{\theta}_{t+1}^E) = \max\left\{0; \mathbb{E}_t\left[\pi_t^{E,prem}(\theta)\Big| f^E \leq f_t^E\right]\right\} - w^N f^r \left[1 - \lambda \frac{Y(f_{t+1}^E)}{Y(f_t^E)}\right] \tag{17}$$

From period 0 up to $\hat{t}$, the strategy of exploring the offshoring potential in the East dominates the exploration in the South. Therefore, the offshoring flow concentrates in the East, while the South remains exclusively specialised in the production of the homogeneous good.

At $\hat{t}$, the beliefs about institutional conditions are:

$$f^S \sim Y(f^S) \text{ with } f^S \in [\underline{f}^S, \bar{f}^S]$$
$$f^E = f^E(\theta_{\hat{t}}^E)$$

with $\theta_{\hat{t}}^E$ denoting the least productive firm offshoring from East in period $\hat{t}$.

Consider $|\delta|$ is large enough[30] such that the following condition holds:

$$\mathcal{D}_{\hat{t}}^S(\theta_{\hat{t}}^E; \bar{\bar{\theta}}^S, \bar{\bar{\theta}}^S) < 0$$

This means that the most productive firm doing domestic sourcing at $\hat{t} + 1$ [31] does not find attractive to explore her offshoring potential in the South. Therefore, there are no informational externalities that will trigger the sequential offshoring flow into the South.

This drives the economy to an inefficient steady state in which the specialisation of countries is not the optimal, and with negative welfare consequences. In other words, the welfare gains from offshoring are not fully realised.

$$\theta_t^S \to \infty \forall t \text{ and } \theta_t^E \downarrow \theta_\infty^E > \theta^{S,*} \Rightarrow P_t \downarrow P_\infty > P^* \Rightarrow Q_t \uparrow Q_\infty < Q^*$$

with $\theta_t^S \to \infty \forall t$ refering to the fact that no firm offshore in South at any period t.

[29]When $f^E - \underline{f}^E \leq (1-\lambda) f^r$ then $\hat{t} \to \infty$.

[30]Equivalently, it is possible to consider that fundamentals in the East are good enough such that the true value does not reveal in the first periods. Therefore, firms sourcing domestically, after the true value f^E is revealed, will not find profitable to explore their offshoring potential in South.

[31]i.e. the firm marginally less productive than the offshoring productivity cutoff in East.

Case C-II: Early explorers shifting path. This is a very special case, in which the economy starts in the bad path and it is pushed towards the good steady state.

This situation comes off only when δ is relatively close to zero and the institutional fundamentals in East are extremely bad. In particular, it takes place when f^E is high enough such that some of the first explorers (in $t = 0$) find unprofitable to offshore in East after paying the sunk cost.

As before, $\tilde{\theta}^E_{t=1}$ indicates the least productive firm that have explored her offshoring potential in the East in period $t = 0$, and $\theta^E_{t=1}$ refers to the least productive firm that remained sourcing from East.

Therefore, *"extremely bad"* fundamentals in East[32] implies that the following condition holds:

$$f^E > f^E(\tilde{\theta}^E_{t=1}) \equiv \tilde{f}^E_{t=1}$$

Those firms with productivity $\theta \in [\tilde{\theta}^E_{t=1}, \theta^E_{t=1})$, who have explored their offshoring potential in East in the period $t = 0$, discovered that it is not profitable for them to source from this country.

In consequence, if $|\delta|$ is small enough such that firms $\theta \in [\tilde{\theta}^E_{t=1}, \theta^E_{t=1})$ find profitable to explore their offshoring potential in South in the period $t = 1$, a sequential offshoring process triggers to the South. Formally, this takes place if the following condition holds:

$$\mathcal{D}_{t=1}(\theta^E_{t=1}; \bar{\bar{\theta}}^S, \bar{\bar{\theta}}^S) > 0$$

Intuitively, this implies that at least the most productive firm among those who have failed offshoring from East, must find profitable to explore the offshoring potential in South.

Thence, after the initial period, the beliefs about the institutional conditions in both foreign countries is represented by:

$$f^E = f^E_{t=1}$$
$$f^S \sim \begin{cases} Y(f^S | f^S \leq f^S_t) & \text{if } \tilde{f}^S_t = f^S_t < f^S_{t-1} \\ f^S_t & \text{if } \tilde{f}^S_t < f^S_t \end{cases}$$

and the firm's decision at any period $t \geq 1$ is characterised by the trade-off function defined in equation (16).

To conclude, once the emergence of informational externalities in the South has been triggered, it leads the economy towards the perfect information steady state.

$$\theta^E_t \rightarrow \{\theta^E_{t=1} \veebar \infty\} \text{ and } \theta^S_t \downarrow \theta^{S,*} \Rightarrow P_t \downarrow P^* \Rightarrow Q_t \uparrow Q^*$$

where $\theta^E_t \rightarrow \{\theta^E_{t=1} \veebar \infty\}$ refers to the fact that firms offshoring in East may or may not decide to relocate at some period $t < \infty$.[33].

[32]Extremely bad with respect to the prior beliefs. This would be the case of extremely optimistic beliefs about eastern institutions.

[33]For the characterisation of the relocation decision of firms who have found profitable to initially remain sourcing from East, see section 3.1.2. In this case this relocation decision involves those firms with productivity $\theta \in (\theta^E_{t=1}, \bar{\bar{\theta}}]$

The production allocation depends on whether relocation takes place or not, considering that some firms may remain sourcing from East. Nevertheless, in the long run, the ”exogenous death shock effect” progressively eliminates the firms sourcing from East.

Regarding welfare, the gains from offshoring are achieved in the long run as the economy converges to the perfect information steady state.

Therefore, in the long run the steady state is characterised by:

$$\theta_t^E \to \infty \text{ and } \theta_t^S \downarrow \theta^{S,*} \Rightarrow P_t \downarrow P^* \Rightarrow Q_t \uparrow Q^*$$

3.2 Final consideration about the multi-country model

First, I summarise the main results of the model, and then I close the section with some additional considerations about how could be the model affected by different scopes of spillovers, particularly the results related to the specialisation patterns of countries. Additionally, I briefly present some policy considerations, specifically those related to the effectiveness of institutional reforms under uncertainty, as well as the role of international institutions in shaping the multinational firms' prior beliefs[34].

Summary of the multi-contry model. The extension to a multi-country world results in a multiple equilibria model, which shows the risks and costs faced by the firms when they explore their offshoring potential among several potential locations. The model shows the importance of the informational spillovers as drivers of the countries' *revealed* comparative advantages.

I have characterised above the cases where the economy reaches the perfect information steady state, in which the production allocation across countries corresponds to the comparative advantages in fundamentals, and the welfare gains from offshoring are fully achieved in the long run.

But I have also found and analysed the cases in which the economy converges to an inefficient steady state (bad equilibrium), leading to a suboptimal specialization of countries, and the welfare gains from offshoring are not fully realised. This shows how important are the informational spillovers in the determination (or *revelation*) of the countries' comparative advantages.

The Figure 6 summarises all the cases, and Propositions 5 and 6 present the main results in terms of countries specialization and welfare implications in the long run, respectively.

Proposition 5 (Countries' sectoral specialization: multiple equilibria)**.** *The steady state of the economy converges*

- *to the specialisation of countries according to fundamentals when*

[34] In other words, the role played by international institutions in the formation of the prior beliefs about institutional conditions in foreign countries

- *the prior beliefs are symmetric and "optimistic". This is achieved in a finite time when the relocation takes place, or in the long run when the relocation does not take place.*
- *the prior beliefs are asymmetric and in favour of the country with best fundamentals.*
- *the prior beliefs are asymmetric and in favour of the country with worst fundamentals, and the prior beliefs are extremly optimistic about institutions in that country (Case C-II).*

- *to an inefficient specialisation of countries when*
  - *the prior beliefs are symmetric and "pessimistic".*
  - *the prior beliefs are asymmetric and in favour of the country with worst fundamentals (except in the Case C-II).*

Proposition 6 (Welfare effects)**.** *In the long run, the welfare gains from offshoring*

- *are fully achieved when*
  - *the prior beliefs are symmetric.*
  - *the prior beliefs are asymmetric and in favour of the country with best fundamentals.*
- *are not fully realised when the prior beliefs are asymmetric and in favour of the country with worst fundamentals (except in the Case C-II).*

Beliefs	**Cases**			**Convergence**				
				$\boldsymbol{\theta^E}$		$\boldsymbol{\theta^S}$	$\boldsymbol{P}$	$\boldsymbol{Q}$
				Without death shock effect	***With death shock effect***			
Symmetric	A Simultaneous exploration of countries	"Optimistic" beliefs		$\theta^{S,*} < \theta^E_\infty < \infty$	$\theta^{S,*} < \theta^E_\infty < \infty$	$\theta^{S,*}$	P^*	Q^*
		"Pessimistic" beliefs	No relocation	$\theta^{S,*} < \theta^E_\infty < \infty$	∞	$\theta^{S,*}$	P^*	Q^*
			Relocation	∞				
Asymmetric	B Coordination in good equilibrium			∞	∞	$\theta^{S,*}$	P^*	Q^*
	C Coordination in bad equilibrium	**I:** Bad equilibrium path		$\theta^{S,*} < \theta^E_\infty < \infty$	$\theta^{S,*} < \theta^E_\infty < \infty$	∞	$P_\infty > P^*$	$Q_\infty < Q^*$
		II: Early explorers shifting path	No relocation	$\theta^{S,*} < \theta^E_\infty < \infty$	∞	$\theta^{S,*}$	P^*	Q^*
			Relocation	∞				

Figure 6: Summary of cases- Convergence conditions

Additional considerations. In the cases of strong sector-specific institutions, or industries more dependent on institutional quality[35], the scope of the informational

[35] For instance, under incomplete contracts, those sectors that more contract dependent. However, I will treat this case in further extension, with a specific model for analysing uncertainty in contractual institutions, i.e. uncertainty in contractual frictions.

spillovers may extend only to a industry-specific effect, and thence it may lead to a sectoral specialization of each country. In other words, the sequential offshoring process in one differentiated sector j is separable from the dynamic of other differentiated industries. However, when the scope of the spillovers is larger, i.e. the externalities spill across sectors, this may lead to a more extensive or across-sectors effect.

Regarding the policy considerations, the model pushes some new questions in the discussion about the effectiveness of the institutional reforms.

It is possible to observe directly from the model that a change in the fundamentals in a country, which has the goal of inducing the multinational firms to place the production of the intermediate inputs in that location, may not have the expected results when the perceived institutions is not sufficiently affected. In other words, when foreign firms do not fully believe in the scope of the reforms and a high uncertainty prevails after the changes.

Furthermore, as the informational spillovers produce an increasing differentiation of countries (in terms of beliefs), the pressure on the reforms to impact on the firms' perceptions is higher over time[36]. This pattern becomes harder to break by policy as the offshoring sequence progresses, and countries become increasingly more differentiated in terms of firms' perceptions.

Moreover, in the cases of the economy converging to a "bad" equilibrium path, for the country which has been disadvantaged by the spillovers but posseses better fundamentals, the entire pressure of the "reform" lies on changing the perception that firms have about its institutions.

In this last regard, it may be more effective for those countries to introduce policies mainly defined as signals which are oriented to change firms' perception, i.e. signals that strongly affect the multinational firms' prior uncertainty about institutional quality in those country.

Handley (2014); Handley and Limão (2017) have analysed the effects of international institutions on reducing the policy uncertainty, particularly trade policy, and the respective effects on the trade flows. Similarly, the access (reputation) of the countries to (at) international institutions such as WTO or ICSID, the participation in RTA or FTA, or the introduction of disputes resolutions mechanisms by international arbitration institutions, well known by multinational firms, and the enforcement of their resolutions, may work as strong signals to induce changes of the prior beliefs that multinational firms may have about those countries.

4 Conclusions and further extensions

Institutions are key drivers of the multinational firms' sourcing decisions, and in consequence defining the comparative advantages of countries and the allocation

[36] In the case of sector-specific spillovers, the countries may exploit that in their favour and develop sector-specific institutions, specially those oriented to relatively new industries, in which informational spillovers have had only a weak effect by that moment.

of production worldwide.

However, sourcing decisions are usually taken facing uncertainty about the institutional fundamentals in foreign locations. This problem is particularly relevant with respect to locations in which firms have no previous experience, but also regarding countries in which the governments have implemented deep institutional reforms and thus firms may face doubts about the true scope of those changes.

In a model with two countries, I showed that firms can exploit informational externalities that emerge from other firms' behaviour, and thus better asses their offshoring potential and progressively adjust their sourcing strategies. These informational spillovers result in a sequential offshoring dynamic led by the most productive firms in the market, which converges to the perfect information steady state of the economy. In consequence, informational externalities allow the economy to progressively overcome the initial inefficiencies produced by the prior uncertainty, and thence fully achieve the welfare gains from offshoring in the long run.

In a multi-country world, in which northern firms can choose among different foreign countries for offshoring, I showed that a selection pattern may emerge.

The existence of informational spillovers results in a multiple equlibria model. Therefore, the prior beliefs and differences in institutional fundamentals across countries may drive the economy to the perfect information equilibrium, or may push the system to a bad steady state. While in the first cases the steady state is characterised in the long run by the perfect information welfare gains and the optimal specialization of countries, in the second cases the economy achieves an inefficient specialization of countries and welfare gains from offshoring are not fully realised.

The latter shows the situation in which informational spillovers drive the offshoring flows to certain locations, becoming one source of the countries' *revealed* comparative advantages. In this regard, this model complements the literature on institutions and comparative advantages (Costinot, 2009; Acemoglu et al., 2007), which focuses in the importance of the institutional fundamentals in the specialization of countries.

The scope of the informational spillovers defines or drives the sectoral specialization of each country. When they are sector-specific, it may lead to a sectoral specialization of the countries. However, when the scope of the spillovers is larger (externalities spill across sectors), this leads to a more extensive effect.

Finally, I remarked some policy considerations that are derived from the model, particularly about the effectiveness of institutional reforms under uncertainty, and the role of international institutions in the formation of the prior beliefs of multinational firms.

I will consider further extensions of the model. The first extension refers to a multi-country model with uncertainty in a context of asymmetric countries in institutional fundamentals ($f^N < f^S < f^E$) and in wages ($w^N > w^S > w^E$). This drives the economy to different order of optimal sourcing strategies in terms of the firms' productivity, in which the least productive ones source domestically, the most productive firms offshore in East exploiting the lowest wages, while the middle productivity firms, who cannot afford the eastern fixed costs, source from South.

On the other hand, the results of this paper may be threatened by relaxing Assumption A.2. In other words, when wages responds to the offshoring flows. Therefore, a second extension of the model consists in a model in which wages responds to offshoring flows, i.e. a situation in which the weight of the differentiated sectors in total expenditure is large enough (or foreign countries small enough) to affect wages. The goal of this extension is to analyse how the responses of wages to the offshoring flows may affect the sequential dynamic of offshoring, and the multiple equilibria of the multi-country model. [37]

Further partial equilibrium analysis that I consider for future extensions are: i) incomplete contracts, and ii) contractual frictions with uncertainty in the degree of contractibility in foreign locations (Antràs and Helpman, 2008), instead of uncertainty in the fixed costs.

Finally, I consider an empirical estimation of the model, which tests the sequential dynamic driven by the informational spillovers and the learning mechanism described above, the scope of the spillovers (sector-specific or across sectors), and the selection patterns (country specialisation driven by spillovers).

[37] As a preliminary result, a relocation trade-off in firms' sourcing decisions may arise. This trade-off reflects the situation in which, as an answer to increasing wages in the first chosen country (e.g. East), the most productive firms currently offshoring in East may start exploring their offshoring potential in South. As a result, additionally to the sequential offshoring equilibrium path shown above, a new sequential dynamic emerges. Firms may explore countries sequentially, and this process is again led by the most productive firms in the market.

References

Acemoglu, D., Antràs, P. and Helpman, E. (2007), 'Contracts and technology adoption', *American Economic Review* **97**(3), 916–943.

Aeberhardt, R., Buono, I. and Fadinger, H. (2014), 'Learning, incomplete contracts and export dynamics: theory and evidence from french firms', *European Economic Review* **68**, 219–249.

Albornoz, F., Pardo, H. F. C., Corcos, G. and Ornelas, E. (2012), 'Sequential exporting', *Journal of International Economics* **88**(1), 17–31.

Alfaro, L. and Charlton, A. (2009), 'Intra-industry foreign direct investment', *American Economic Review* **99**(5), 2096–2119.

Antràs, P. and Chor, D. (2013), 'Organizing the global value chain', *Econometrica* **81**(6), 2127–2204.

Antràs, P. and Helpman, E. (2004), 'Global sourcing', *Journal of political Economy* **112**(3), 552–580.

Antràs, P. and Helpman, E. (2008), *Contractual Frictions and Global Sourcing*, Harvard University Press, Cambridge, MA, pp. 9–54.

Antràs, P. and Yeaple, S. R. (2014), Multinational firms and the structure of international trade, *in* 'Handbook of international economics', Vol. 4, Elsevier, pp. 55–130.

Araujo, L., Mion, G. and Ornelas, E. (2016), 'Institutions and export dynamics', *Journal of International Economics* **98**, 2–20.

Carballo, J. (2016), 'Global sourcing under uncertainty'.

Costinot, A. (2009), 'On the origins of comparative advantage', *Journal of International Economics* **77**(2), 255–264.

DeGroot, M. H. (2005), *Optimal statistical decisions*, Vol. 82, John Wiley & Sons.

Grossman, G. M. and Rossi-Hansberg, E. (2008), 'Trading tasks: A simple theory of offshoring', *American Economic Review* **98**(5), 1978–97.

Handley, K. (2014), 'Exporting under trade policy uncertainty: Theory and evidence', *Journal of international Economics* **94**(1), 50–66.

Handley, K. and Limão, N. (2017), 'Policy uncertainty, trade, and welfare: Theory and evidence for china and the united states', *American Economic Review* **107**(9), 2731–83.

Helpman, E. (2006), 'Trade, fdi, and the organization of firms', *Journal of economic literature* **44**(3), 589–630.

Kohler, W., Kukharskyy, B. et al. (2018), Offshoring under uncertainty, Technical report, CESifo Group Munich.

Levchenko, A. A. (2007), 'Institutional quality and international trade', *The Review of Economic Studies* **74**(3), 791–819.

Melitz, M. J. (2003), 'The impact of trade on intra-industry reallocations and aggregate industry productivity', *Econometrica* **71**(6), 1695–1725.

Nguyen, D. X. (2012), 'Demand uncertainty: Exporting delays and exporting failures', *Journal of International Economics* **86**(2), 336–344.

Nunn, N. (2007), 'Relationship-specificity, incomplete contracts, and the pattern of trade', *The Quarterly Journal of Economics* **122**(2), 569–600.

Ramondo, N., Rappoport, V. and Ruhl, K. J. (2013), 'The proximity-concentration tradeoff under uncertainty', *Review of Economic Studies* **80**(4), 1582–1621.

Ramondo, N., Rodríguez-Clare, A. and Tintelnot, F. (2015), 'Multinational production: Data and stylized facts', *American Economic Review* **105**(5), 530–36.

Rob, R. (1991), 'Learning and capacity expansion under demand uncertainty', *The Review of Economic Studies* **58**(4), 655–675.

Rob, R. and Vettas, N. (2003), 'Foreign direct investment and exports with growing demand', *The Review of Economic Studies* **70**(3), 629–648.

Segura-Cayuela, R. and Vilarrubia, J. M. (2008), 'Uncertainty and entry into export markets'.

Stokey, N. L. and Lucas, R. J. (1989), *Recursive methods in economic dynamics*, Harvard University Press.

Sutton, R. S. and Barto, A. G. (2018), *Reinforcement learning: An introduction*, MIT press.

A Complete Contracts - Perfect information model

A.1 Consumer's problem

To obtain the variety i demand function $q_j(i)$, I maximize the utility subject to the following budget constraint:

$$p_0 q_0 + \sum_{j=1}^{J} \int_{i \in I_j} p_j(i) q_j(i) di \leq E$$

From the FOCs for two different varieties i, i' in sector j:

$$\left[\frac{q_j(i)}{q_j(i')}\right]^{\alpha_j - 1} = \frac{p_j(i)}{p_j(i')} \Leftrightarrow \quad q_j(i) = \left[\frac{p_j(i')}{p_j(i)}\right]^{\frac{1}{1-\alpha_j}} q_j(i')$$

Given the Cobb-Douglas utility function, $(\gamma_j)E$ refers to the expenditure in differentiated sector j's goods. Plugging the expression above for $q_j(i)$ into the budget constraint:

$$\gamma_j E = \int_{i \in I_j} p_j(i) q_j(i) di \quad \Leftrightarrow q_j(i') = \frac{\gamma_j E}{P_j} \left[\frac{p_j(i')}{P_j}\right]^{-\sigma_j}$$

This expression holds for any variety i, thus

$$q_j(i) = \frac{\gamma_j E}{P} \left[\frac{p_j(i)}{P_j}\right]^{-\sigma_j}$$

Or equivalently, from the FOCs, I can obtain:

$$q_j(i) = \left[\gamma_j E Q_j^{-\alpha} p_j(i)^{-1}\right]^{\sigma_j}$$

To conclude, the demand for homogenous good q_0 is given by

$$q_0 = \frac{\gamma_0 E}{p_0}$$

A.2 Producers' problem

The per-period revenues of a firm producing a variety i is given by:

$$r_j(i) = p_j(i) q_j(i)$$

Plugging the expression from equation (3), and replacing with the production function (5):

$$r_j(i) = \gamma_j E Q_j^{-\alpha} q_j(i)^{\alpha_j}$$

$$\Rightarrow r_j(i) = \gamma_j E Q_j^{-\alpha_j} \left[\theta \left(\frac{x_{h,j}(i)}{\eta_j}\right)^{\eta_j} \left(\frac{x_{m,j}(i)}{1-\eta_j}\right)^{1-\eta_j}\right]^{\alpha_j}$$

Solution to producer's problem. Given that all investments are contractible, the final good producer solves the following optimization problem.

$$\max_{x_{h,j}(i),x_{m,j}(i)} \pi_j = r_j(i) - w^N x_{h,j}(i) - w^l x_{m,j}(i) - w^N f_j^l$$

where $l = \{N, S\}$ refers to the location of the input's supplier.

By solving the FOCs,

$$x_{h,j}(i) = \frac{\alpha_j \eta_j}{w^N} r_j(i)$$

$$x_{m,j}(i) = \frac{\alpha_j (1-\eta_j)}{w^l} r_j(i)$$

Dividing the two equations above, and plugging it into the FOCs, the optimal HQ's investments are:

$$x^*_{h,j}(i) = \frac{\alpha_j \eta_j}{w^N} r_j^{l,*}(\theta) \tag{18}$$

with $r_j^{l,*}(\theta)$ given by:

$$r_j^{l,*}(\theta) \equiv \alpha_j^{\sigma_j - 1} \theta^{\sigma_j - 1} (\gamma_j E)^{\sigma_j} Q_j^{1-\sigma_j} \left[(w^N)^{\eta_j} (w^l)^{1-\eta_j} \right]^{1-\sigma_j} \tag{19}$$

Equivalently, the optimal supplier's investments are:

$$x^*_{m,j}(i) = \frac{\alpha_j (1-\eta_j)}{w^l} r_j^{l,*}(\theta) \tag{20}$$

Plugging the optimal investments into (5), I get the optimal production for a firm with productivity θ:

$$q_j^*(i) = \theta^{\sigma_j} \alpha_j^{\sigma_j} (\gamma_j E)^{\sigma_j} Q_j^{1-\sigma_j} \left[(w^N)^{\eta_j} (w^l)^{1-\eta_j} \right]^{-\sigma_j} \tag{21}$$

Consequently, the optimal price for a variety produced by a firm with productivity θ with a supplier from location l is:

$$p_j^*(i) = \theta^{-1} \alpha_j^{-1} (w^N)^{\eta_j} (w^l)^{1-\eta_j}$$

Finally, the profits realised by a firm with productivity θ for each sourcing strategy, i.e. domestic sourcing and offshoring, are:

$$\pi_j^l(\theta, Q_j, \eta_j, f_j^l, w^l) = r_j^{l,*}(\theta) - w^N x^*_{h,j}(i) - w^l x^*_{m,j}(i) - w^N f_j^l$$

Replacing with the expressions above for optimal investments:

$$\pi_j^l(\theta, .) = \frac{r_j^{l,*}(\theta)}{\sigma_j} - w^N f_j^l$$

Therefore, plugging the solution for revenues,

$$\pi_j^l(\theta, .) = \theta^{\sigma_j - 1} (\gamma_j E)^{\sigma_j} Q_j^{1-\sigma_j} \psi_j^l - w^N f_j^l \tag{22}$$

with $l = \{N, S\}$, and ψ_j^l is defined as:

$$\psi_j^l \equiv \frac{\alpha_j^{\sigma_j - 1}}{\sigma_j \left[(w^N)^{\eta_j} (w^l)^{1-\eta_j} \right]^{\sigma_j - 1}}$$

B Initial conditions: Non-tradable intermediate inputs ($n.t.i.$)

I focus the analysis in one differentiated sector, therefore I drop the subscript j.

B.1 Aggregation: Open Economy with n.t.i.

The production, price and per-period profits for a firm with productivity θ in the steady state of the n.t.i. economy are given by:

$$q_t^{n.t.i.}(\theta) = \left(\frac{\theta\alpha(1-\gamma_0)E(Q^{n.t.i.})^{-\alpha_j}}{w^N} \right)^{\sigma} \tag{23}$$

$$p_t^{n.t.i.}(\theta) = \frac{w^N}{\alpha\theta} \tag{24}$$

$$\pi_t^N(.) = \theta^{\sigma-1}[(1-\gamma_0)E]^{\sigma}(Q^{n.t.i.})^{1-\sigma}\psi^N - w^N f^N \tag{25}$$

with $\psi^N \equiv \sigma^{-1}\left[\frac{\alpha}{w^N}\right]^{\sigma-1}$.

B.1.1 Sectoral price index

The price index can be represented as:

$$P^{n.t.i.} = \left[\int_{i\in I} p(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} \quad \Leftrightarrow \quad P^{n.t.i.} = \left[\int_0^{\infty} p(\theta)^{1-\sigma} H^{n.t.i.}\mu(\theta)d\theta \right]^{\frac{1}{1-\sigma}} \tag{26}$$

where $H^{n.t.i.}$ refers to the total number of final-good producers active in the market in this sector, and $\mu(\theta)$ denotes the ex-post distribution of firm productivities in the market.

$$\mu(\theta) = \begin{cases} \frac{g(\theta)}{1-G(\underline{\theta}^{n.t.i.})} & \text{if } \theta \geq \underline{\theta}^{n.t.i.} \\ 0 & \text{if } \theta < \underline{\theta}^{n.t.i.} \end{cases} \tag{27}$$

By plugging equation (24) into (26), I get the price index of the differentiated sector in terms of the average productivity in that sector.

$$P^{n.t.i.} = (H^{n.t.i.})^{\frac{1}{1-\sigma}} \frac{w^N}{\alpha} \left[\left(\int_0^{\infty} \theta^{\sigma-1}\mu(\theta)d\theta \right)^{\frac{1}{\sigma-1}} \right]^{-1}$$

Defining $\bar{\theta}^{n.t.i.}$ as the average productivity in the sector:

$$\bar{\theta}^{n.t.i.} \equiv \left(\int_0^{\infty} \theta^{\sigma-1}\mu(\theta)d\theta \right)^{\frac{1}{\sigma-1}} = \left(\frac{1}{1-G(\underline{\theta}^{n.t.i.})} \int_{\underline{\theta}^{n.t.i.}}^{\infty} \theta^{\sigma-1}g(\theta)d\theta \right)^{\frac{1}{\sigma-1}} \tag{28}$$

Replacing the equation (28) into the price index,

$$P^{n.t.i.} = (H^{n.t.i.})^{\frac{1}{1-\sigma}} \frac{w^N}{\alpha\bar{\theta}^{n.t.i.}} \quad \Rightarrow \quad P^{n.t.i.} = (H^{n.t.i.})^{\frac{1}{1-\sigma}} p(\bar{\theta}^{n.t.i.}) \tag{29}$$

B.1.2 Sectoral aggregate consumption

The aggregate consumption in terms of the quantities produced by the average active firm is given by

$$Q^{n.t.i.} = \left[\int_{i\in I} q(i)^{\alpha} di\right]^{1/\alpha} \quad \Leftrightarrow \quad Q^{n.t.i.} = \quad \left[\int_0^{\infty} q(\theta)^{\frac{\sigma-1}{\sigma}} H^{n.t.i.}\mu(\theta)d\theta\right]^{\frac{\sigma}{\sigma-1}}$$

$$Q^{n.t.i.} = (H^{n.t.i.})^{\frac{\sigma}{\sigma-1}} \left[\frac{\alpha(1-\gamma_0)E}{w^N}\right]^{\sigma} (Q^{n.t.i.})^{1-\sigma} \left[\int_0^{\infty} \theta^{\sigma-1}\mu(\theta)d\theta\right]^{\frac{\sigma}{\sigma-1}}$$

$$Q^{n.t.i.} = (H^{n.t.i.})^{\frac{\sigma}{\sigma-1}} q(\bar{\theta}^{n.t.i.}) \quad \Rightarrow \quad Q^{n.t.i.} = (H^{n.t.i.})^{\frac{1}{\sigma-1}} \frac{\alpha(1-\gamma_0)E}{w^N} \bar{\theta}^{n.t.i.} \tag{30}$$

B.1.3 Zero Cutoff Profit Condition (ZCPC)

The firm's value function is:

$$v^{n.t.i.}(\theta) = \max\left\{0; v^{N,n.t.i.}(\theta)\right\}$$

with

$$v^{N,n.t.i.}(\theta) = \max\left\{0; \sum_{t=0}^{\infty} \lambda^t \pi^{N,n.t.i.}(\theta)\right\} = \max\left\{0; \frac{\pi^{N,n.t.i.}(\theta)}{1-\lambda}\right\}$$

where λ refers to the per period survival probability to an exogenous bad shock.

Thence, using the zero cutoff profit condition (ZCPC), the market productivity cutoff, denoted as $\underline{\theta}^*$, is implicitly defined by $\pi^{N,n.t.i.}(\underline{\theta}^{n.t.i.}) = 0$. Thus, solving this expression for $\underline{\theta}^{n.t.i.}$, I get:

$$\Pi^N = 0 \Leftrightarrow \frac{\pi_t^N}{1-\lambda} = 0 \Leftrightarrow \pi_t^N = 0$$

$$\Leftrightarrow \underline{\theta}^{n.t.i.} = [(1-\gamma_0)E]^{\frac{\sigma}{1-\sigma}} Q^{n.t.i.} \left[\frac{w^N f^N}{\psi^N}\right]^{\frac{1}{\sigma-1}} \tag{31}$$

Also, by using the (ZCPC), I get the revenue level for the cutoff productivity firm $r^{N,n.t.i.}(\underline{\theta}^{n.t.i.})$.

$$\pi_t^N(\underline{\theta}^{n.t.i.}) = 0 \quad \Rightarrow \quad r^N(\underline{\theta}^{n.t.i.}) = \sigma w^N f^N \tag{32}$$

Furthermore, the revenues of the average firm as a function of the cutoff firm revenues is given by:

$$\frac{r^N(\bar{\theta}^{n.t.i.})}{r^N(\underline{\theta}^{n.t.i.})} = \left(\frac{\bar{\theta}^{n.t.i.}}{\underline{\theta}^{n.t.i.}}\right)^{\sigma-1} \quad \Rightarrow \quad r^N(\bar{\theta}^{n.t.i.}) = \left(\frac{\bar{\theta}^{n.t.i.}}{\underline{\theta}^{n.t.i.}}\right)^{\sigma-1} r^N(\underline{\theta}^{n.t.i.}) \tag{33}$$

and the average revenues are:

$$\bar{r}^{n.t.i.} \equiv r^N(\bar{\theta}^{n.t.i.}) = \left(\frac{\bar{\theta}^{n.t.i.}}{\underline{\theta}^{n.t.i.}}\right)^{\sigma-1} \sigma w^N f^N \tag{34}$$

Finally, it is possible to obtain the profits of the average firm as:

$$\bar{\pi}^{n.t.i.} \equiv \pi^N(\bar{\theta}^{n.t.i.}) = \frac{r^N(\bar{\theta}^{n.t.i.})}{\sigma} - w^N f^N$$

Replacing with (34) and plugging the cutoff revenues from (32), I obtain the (ZCPC):

$$\bar{\pi}^{n.t.i.} \equiv \pi^N(\bar{\theta}^{n.t.i.}) = w^N f^N \left[\left(\frac{\bar{\theta}^{n.t.i.}}{\underline{\theta}^{n.t.i.}} \right)^{\sigma-1} - 1 \right] \tag{35}$$

B.1.4 Free Entry Condition (FEC)

All the active final good producers, except for the cutoff firm $\underline{\theta}^{n.t.i.}$, earn positive profits. Therefore, $\bar{\pi}^{n.t.i.} > 0$. Given this expected positive profits, firms decide to sink the entry cost f_e and entry into the market.

The present value of a firm, conditional on sucessful entry, is:

$$\bar{v} = \int_{\underline{\theta}^{n.t.i.}}^{\infty} v(\theta)\mu(\theta)d\theta = \frac{\bar{\pi}^{n.t.i.}}{1-\lambda}$$

On the other hand, the net value of entry is given by:

$$v_e = p_{in}\bar{v} - w^N f_e = \frac{1 - G(\underline{\theta}^{n.t.i.})}{1-\lambda}\bar{\pi}^{n.t.i.} - w^N f_e$$

By (FEC): $v_e = 0$. Therefore,

$$\bar{\pi}^{n.t.i.} = \frac{(1-\lambda)f_e w^N}{1 - G(\underline{\theta}^{n.t.i.})} \tag{36}$$

B.1.5 Equilibrium: Number of firms

From (ZCPC) and (FEC):

$$\bar{\theta}^{n.t.i.} = \left[\frac{(1-\lambda)f_e}{[1 - G(\underline{\theta}^{n.t.i.})]f^N} + 1 \right]^{\frac{1}{\sigma-1}} \underline{\theta}^{n.t.i.} \tag{37}$$

The number of active firms is given by:

$$H^{n.t.i.} = \frac{R^{n.t.i.}}{\bar{r}^{n.t.i.}} \Leftrightarrow H^{n.t.i.} = \frac{(1-\gamma_0)E}{\bar{r}^{n.t.i.}}$$

Using $\bar{r}^{n.t.i.} = \sigma\Big[\bar{\pi}^{n.t.i.} + w^N f^N\Big]$, the number of active firms in sector j is:

$$H^{n.t.i.} = \frac{(1-\gamma_0)E}{\sigma\left[\bar{\pi}^{n.t.i.} + w^N f^N\right]}$$

and replacing $\bar{\pi}^{n.t.i.}$ with (ZCPC), the number of active firms is:

$$H^{n.t.i.} = \frac{(1-\gamma_0)E}{\sigma w^N f^N} \left(\frac{\underline{\theta}^{n.t.i.}}{\bar{\theta}^{n.t.i.}} \right)^{\sigma-1} \tag{38}$$

C Perfect Information: Tradable intermediate inputs

The revenues for a firm with productivity θ doing domestic sourcing is represented as $r^{N,*}(\theta)$. Instead, when the firm chooses offshoring the revenue is denoted as $r^{S,*}(\theta)$. Dividing both expressions:

$$\frac{r^{S,*}(\theta)}{r^{N,*}(\theta)} = \left(\frac{w^N}{w^S}\right)^{(1-\eta)(\sigma-1)} \Leftrightarrow r^{S,*}(\theta) = \left(\frac{w^N}{w^S}\right)^{(1-\eta)(\sigma-1)} r^{N,*}(\theta)$$

Substracting on both sides $r^{N,*}(\theta)$, I obtain the offshoring premium in revenues received by a firm with productivity θ, when the firm decides for offshoring.

$$r^{S,prem}(\theta) \equiv r^{S,*}(\theta) - r^{N,*}(\theta) = \left[\left(\frac{w^N}{w^S}\right)^{(1-\eta)(\sigma-1)} - 1\right] r^{N,*}(\theta) \tag{39}$$

Equivalently, the per period offshoring premium in profits for a firm with productivity θ (without considering the market research sunk cost) is given by:

$$\pi^{S,prem}(\theta) \equiv \pi^S(\theta) - \pi^N(\theta)$$

$$\pi^{S,prem}(\theta) = \frac{\alpha^{\sigma-1}\theta^{\sigma-1}[(1-\gamma_0)E]^\sigma Q^{1-\sigma}}{\sigma} \left[\frac{(w^N)^{(1-\eta)(\sigma-1)} - (w^S)^{(1-\eta)(\sigma-1)}}{\left[(w^S)^{(1-\eta)} w^N\right]^{(\sigma-1)}}\right] - w^N \left[f^S - f^N\right]$$

Thus, the per period offshoring premium in profits for a firm with productivity θ, without considering the market reserach sunk cost, can be equivalently expressed as[38]:

$$\Leftrightarrow \pi^{S,prem}(\theta) = \frac{r^{N,*}(\theta)}{\sigma} \left[\left(\frac{w^N}{w^S}\right)^{(1-\eta)(\sigma-1)} - 1\right] - w^N \left[f^S - f^N\right] \tag{40}$$

Let's define $\bar{\theta}^S$ as the average productivity of the firms doing offshoring. Formally,

$$\bar{\theta}^S \equiv \left[\frac{1}{1-G(\theta^{S,*})} \int_{\theta^{S,*}}^{\infty} \theta^{\sigma-1} g(\theta) d\theta\right]^{\frac{1}{\sigma-1}} \tag{41}$$

On the other hand, the variable $\bar{\theta}$ is still defined as:

$$\bar{\theta} \equiv \left(\int_0^\infty \theta^{\sigma-1}\mu(\theta)d\theta\right)^{\frac{1}{\sigma-1}} = \left(\frac{1}{1-G(\underline{\theta})} \int_{\underline{\theta}}^{\infty} \theta^{\sigma-1} g(\theta) d\theta\right)^{\frac{1}{\sigma-1}} \tag{42}$$

The *light* area of Figure 2, below the $\pi^N(\theta)$ function, can be computed in a similar way as in the case where domestic sourcing was the only available option:

$$\pi^N(\bar{\theta}) = w^N f^N \left[\left(\frac{\bar{\theta}}{\underline{\theta}^*}\right)^{\sigma-1} - 1\right]$$

[38]It is straightforward to see that this offshoring profit premium can be positive or negative depending on the productivity level of the firm.

On the other hand, the per period offshoring premium in profits, without considering the offshoring market research sunk cost, of the average productivity firm in offshoring is represented by:

$$\pi^{S,prem}(\bar{\theta}^S) \equiv \pi^S(\bar{\theta}^S) - \pi^N(\bar{\theta}^S)$$

with the aggregate offshoring profit premium if the *dark* area in Figure 2 between both profit functions.

Replacing in the previous equation with the respective profit equations evaluated at $\bar{\theta}^S$:

$$\Leftrightarrow \pi^{S,prem}(\bar{\theta}^S) = \frac{r^{N,*}(\bar{\theta}^S)}{\sigma}\left[\left(\frac{w^N}{w^S}\right)^{(1-\eta)(\sigma-1)} - 1\right] - w^N\left[f^S - f^N\right] \tag{43}$$

Therefore, the average per-period profits when the intermediate inputs become tradable are given by:

$$\begin{aligned}\bar{\pi} &= \pi^N(\bar{\theta}) + p_{\text{off}}\left[\pi^{S,prem}(\bar{\theta}^S) - (1-\lambda)w^N f^r\right] \\ &= w^N f^N\left[\left(\frac{\bar{\theta}}{\underline{\theta}^*}\right)^{\sigma-1} - 1\right] + \chi^*\left[\pi^{S,prem}(\bar{\theta}^S) - (1-\lambda)w^N f^r\right]\end{aligned} \tag{44}$$

with $\chi^* \equiv \frac{1-G(\theta^{S,*})}{1-G(\underline{\theta}^*)}$ denoting the share of offshoring firms. The first term of the RHS refers to the average profits obtain by the firms if they would all have choosen domestic sourcing, while the second term denotes the premium in profits received by those firms that decide to offshore adjusted by the share of offshoring firms among the active ones.

Equivalently, the average revenue is given by:

$$\begin{aligned}\bar{r} &= r^N(\bar{\theta}) + \chi^*\left[r^S(\bar{\theta}^S) - r^N(\bar{\theta}^S)\right] \\ &= r^N(\bar{\theta}) + \chi^*\left[\left(\frac{w^N}{w^S}\right)^{(\sigma-1)(1-\eta)} - 1\right] r^N(\bar{\theta}^S)\end{aligned} \tag{45}$$

Finally, the offshoring profit premium for the firm with the offshoring productivity cutoff:

$$\begin{aligned}&\pi^{S,prem}(\theta^{S,*}) - (1-\lambda)w^N f^r = 0 \\ \Rightarrow &r^{N,*}(\theta^{S,*}) = \sigma w^N\left[f^S + (1-\lambda)f^r - f^N\right]\left[\left(\frac{w^N}{w^S}\right)^{(1-\eta)(\sigma-1)} - 1\right]^{-1}\end{aligned}$$

Dividing by the revenues of the firm at the market cutoff productivity level:

$$\frac{r^{N,*}(\theta^{S,*})}{r^{N,*}(\underline{\theta}^*)} = \left(\frac{f^S + (1-\lambda)f^r}{f^N} - 1\right)\left[\left(\frac{w^N}{w^S}\right)^{(1-\eta)(\sigma-1)} - 1\right]^{-1}$$

Also, by using the equivalent of equation (33), it is possible to show

$$\frac{r^{N,*}(\theta^{S,*})}{r^{N,*}(\underline{\theta}^*)} = \left(\frac{\theta^{S,*}}{\underline{\theta}^*}\right)^{\sigma-1} \tag{46}$$

Putting both equations together, and solving for the offshoring productivity cutoff:

$$\left(\frac{\theta^{S,*}}{\underline{\theta}^*}\right)^{\sigma-1} = \left(\frac{f^S + (1-\lambda)f^r}{f^N} - 1\right)\left[\left(\frac{w^N}{w^S}\right)^{(1-\eta)(\sigma-1)} - 1\right]^{-1}$$

$$\Rightarrow \theta^{S,*} = \left(\frac{f^S + (1-\lambda)f^r}{f^N} - 1\right)^{\frac{1}{\sigma-1}}\left[\left(\frac{w^N}{w^S}\right)^{(1-\eta)(\sigma-1)} - 1\right]^{\frac{1}{1-\sigma}}\underline{\theta}^* \tag{47}$$

C.1 Price index in sector j

The price of a variety i produced by a firm with productivity θ which source only domestically is given by:

$$p(\theta) = \frac{w^N}{\alpha\theta} \tag{48}$$

Meanwhile the price of a variety i produced by a firm with productivity θ which offshore is:

$$p^{\text{off}}(\theta) = \frac{(w^N)^\eta (w^S)^{1-\eta}}{\alpha\theta} \tag{49}$$

By substracting the equation (48) from (49), I get the price differential of an offshoring firm with productivity θ:

$$p^{\text{off}}(\theta) - p(\theta) = \frac{(w^N)^\eta\left[(w^S)^{1-\eta} - (w^N)^{1-\eta}\right]}{\alpha\theta} \tag{50}$$

If $w^S < w^N$, as defined by Assumption A.2, thence $p^{\text{off}}(\theta) - p(\theta) < 0$, i.e. offshoring firms can charge a lower price for a given productivity θ.

Moreover, the offshoring price of a firm with productivity θ as a function of its domestic sourcing price is given by:

$$p^{\text{off}}(\theta) = \left(\frac{w^S}{w^N}\right)^{1-\eta} p(\theta) \tag{51}$$

I define P^{off} as the price index of the firms doing offshoring, and $P^{\text{off}|n.t.i}$ as the price index of the same firms doing offshoring but computed under the cost structure of domestic sourcing. Formally, they are defined:

$$P^{\text{off}} \equiv \left[\int_{\theta^{S,*}}^{\infty} [p^{\text{off}}(\theta)]^{1-\sigma} H \frac{g(\theta)}{1 - G(\theta^{S,*})} d\theta\right]^{\frac{1}{1-\sigma}} \tag{52}$$

$$P^{\text{off}|n.t.i} \equiv \left[\int_{\theta^{S,*}}^{\infty} [p(\theta)]^{1-\sigma} H \frac{g(\theta)}{1 - G(\theta^{S,*})} d\theta\right]^{\frac{1}{1-\sigma}} \tag{53}$$

Finally, to obtain the sectoral price index:

$$P^{1-\sigma} = \int_{\underline{\theta}^*}^{\theta^{S,*}} p(\theta)^{1-\sigma} H \frac{g(\theta)}{1 - G(\underline{\theta}^*)} d\theta + \int_{\theta^{S,*}}^{\infty} [p^{\text{off}}(\theta)]^{1-\sigma} H \frac{g(\theta)}{1 - G(\underline{\theta}^*)} d\theta$$

$$
\begin{aligned}
P^{1-\sigma} = & \int_{\underline{\theta}^*}^{\infty} p(\theta)^{1-\sigma} H \frac{g(\theta)}{1-G(\underline{\theta}^*)} d\theta \\
& + \frac{1-G(\theta^{S,*})}{1-G(\underline{\theta}^*)} \int_{\theta^{S,*}}^{\infty} [p^{\text{off}}(\theta)]^{1-\sigma} H \frac{g(\theta)}{1-G(\theta^{S,*})} d\theta \\
& - \frac{1-G(\theta^{S,*})}{1-G(\underline{\theta}^*)} \int_{\theta^{S,*}}^{\infty} p(\theta)^{1-\sigma} H \frac{g(\theta)}{1-G(\theta^{S,*})} d\theta
\end{aligned}
$$

Therefore, the price index is

$$
\Rightarrow P^{1-\sigma} = \left(P^{\text{n.t.i.}}\right)^{1-\sigma} + \chi^* \left[\left(P^{\text{off}}\right)^{1-\sigma} - \left(P^{\text{off|n.t.i.}}\right)^{1-\sigma} \right]
$$

Furthermore, using equation (51), the sectoral price index for the tradable intermediate input economy, P, is given by the following expression:

$$
P^{1-\sigma} = \left(P^{\text{n.t.i.}}\right)^{1-\sigma} + \chi^* \left[\left(\frac{w^N}{w^S}\right)^{(1-\eta)(\sigma-1)} - 1 \right] \left(P^{\text{off|n.t.i.}}\right)^{1-\sigma} \tag{54}
$$

It is possible to observe that the price index is increasing in southern wages, i.e. $\partial P/\partial w^S > 0$. Moreover, given $w^S < w^N$, the price index is increasing in the offshoring cutoff $\theta^{S,*}$. Therefore, reductions in the offshoring productivity cutoff, i.e. more firms choosing offshore, leads to reductions in the price index of that sector.

Moreover, as $\theta^{S,*} \to \infty$, the share of offshoring firms goes to zero, i.e. $\chi^* \to 0$. Therefore, the second term of the RHS of the equation (54) vanishes and the first term shows $P^{n.t.i.}(\underline{\theta}^*) \uparrow P^{n.t.i}(\underline{\theta}^{n.t.i})$ and $\underline{\theta}^* \downarrow \underline{\theta}^{n.t.i.}$. In other words, $P \downarrow P^{n.t.i.}$, where the last corresponds to the price index of the $n.t.i.$ model.

C.2 Aggregate consumption in sector j

Using the relation $Q = \frac{(1-\gamma_0)E}{P}$, [39]and the price index from equation (54), the sectoral aggregate consumption is:

$$
Q = (1-\gamma_0)E \left[\left(P^{\text{n.t.i.}}\right)^{1-\sigma} + \chi^* \left[\left(\frac{w^N}{w^S}\right)^{(1-\eta)(\sigma-1)} - 1 \right] \left(P^{\text{off|n.t.i.}}\right)^{1-\sigma} \right]^{\frac{1}{\sigma-1}} \tag{55}
$$

As expected, the sectoral aggregate consumption is decreasing in both, southern wages and the offshoring productivity cutoff. As before, the latter implies that more firms choosing offshoring leads to a higher sectoral aggregate consumption.

C.3 Firm entry and exit

I derive now the (ZCPC) and (FEC) for the economy with tradable intermediate input.

[39] As I focus the analysis in one differentiated sector, thus I drop the subscript j, I assume that there is only one differentiated sector in the economy. Therefore, the expenditure share in the differentiated sector under analysis is given by $1-\gamma_0$, i.e. by the expenditure that is not intended for consumption of the homogeneous good. This is a trivial assumption which I do in order to simplify notation.

C.3.1 Zero Cutoff Profit Condition (ZCPC)

The firm's value function is still represented by the same function:

$$v(\theta) = \max\left\{0; v^l(\theta)\right\}$$

with

$$v^l(\theta) = \max\left\{0; \sum_{t=0}^{\infty} \lambda^t \pi^l(\theta)\right\} = \max\left\{0; \frac{\pi^l(\theta)}{1-\lambda}\right\}$$

As before, the market productivity cutoff denoted as $\underline{\theta}^*$ is implicitly defined by the zero cutoff profit condition (ZCPC), $\pi^N(\underline{\theta}^*) = 0$. Solving this expression for $\underline{\theta}^*$, the market productivity cutoff is:

$$\underline{\theta}^* = [(1-\gamma_0)E]^{\frac{\sigma}{1-\sigma}} Q \left[\frac{w^N f^N}{\psi^N}\right]^{\frac{1}{\sigma-1}} \tag{56}$$

As before, from the (ZCPC) I get the same expression (32). Thence, dividing $\bar{r}$ from equation (45) by the cutoff firm's revenues (32), I can express the average revenues as a function of the cutoff firm's revenues:

$$\frac{\bar{r}}{r(\underline{\theta}^*)} = \frac{r^N(\bar{\theta})}{r(\underline{\theta}^*)} + \chi^* \left[\left(\frac{w^N}{w^S}\right)^{(\sigma-1)(1-\eta)} - 1\right] \frac{r^N(\bar{\theta}^S)}{r(\underline{\theta}^*)}$$

Replacing the first term and second terms of the RHS by equivalent expressions from equation (33),

$$\frac{\bar{r}}{r(\underline{\theta}^*)} = \left(\frac{\bar{\theta}}{\underline{\theta}^*}\right)^{\sigma-1} + \chi^* \left[\left(\frac{w^N}{w^S}\right)^{(\sigma-1)(1-\eta)} - 1\right] \left(\frac{\bar{\theta}^S}{\underline{\theta}^*}\right)^{\sigma-1}$$

Solving for $\bar{r}$, and replacing $r(\underline{\theta}^*)$ with its expression from equation (32):

$$\bar{r} = \left[\left(\frac{\bar{\theta}}{\underline{\theta}^*}\right)^{\sigma-1} + \chi^* \left[\left(\frac{w^N}{w^S}\right)^{(\sigma-1)(1-\eta)} - 1\right] \left(\frac{\bar{\theta}^S}{\underline{\theta}^*}\right)^{\sigma-1}\right] \sigma w^N f^N \tag{57}$$

Taking the average profits from equation (44), and plugging it into equation (43):

$$\begin{aligned}
\bar{\pi} =& \pi^N(\bar{\theta}) + \chi^* \left[\pi^{S,prem}(\bar{\theta}^S) - (1-\lambda) w^N f^r\right] \\
=& w^N f^N \left[\left(\frac{\bar{\theta}}{\underline{\theta}^*}\right)^{\sigma-1} - 1\right] + \chi^* \frac{r^{N,*}(\bar{\theta}^S)}{\sigma} \left[\left(\frac{w^N}{w^S}\right)^{(1-\eta)(\sigma-1)} - 1\right] \\
& - \chi^* w^N \left[f^S + (1-\lambda) f^r - f^N\right]
\end{aligned}$$

Finally, replacing $r^{N,*}(\bar{\theta}^S)$, the (ZCPC) is given by:

$$\begin{aligned}
\bar{\pi} =& w^N f^N \left[\left[\left(\frac{\bar{\theta}}{\underline{\theta}^*}\right)^{\sigma-1} - 1\right] + \chi^* \left[\left(\frac{w^N}{w^S}\right)^{(1-\eta)(\sigma-1)} - 1\right] \left(\frac{\bar{\theta}^S}{\underline{\theta}^*}\right)^{\sigma-1}\right] \\
& - \chi^* w^N \left[f^S + (1-\lambda) f^r - f^N\right]
\end{aligned} \tag{58}$$

C.3.2 Free Entry Condition (FEC)

The (FEC) is given by the following expression:

$$v_e = p_{\text{in}} \frac{\bar{\pi}}{1-\lambda} - w^N f_e = 0 \quad \Rightarrow \quad \bar{\pi} = \frac{(1-\lambda) w^N f_e}{1 - G(\underline{\theta}^*)} \tag{59}$$

C.3.3 Sectoral equilibrium. Number of firms.

As before, putting the (ZCPC) and (FEC) together, I can obtain the sectoral equilibrium productivity cutoff and the average profits in the sector.

From the (ZCPC) and (FEC), I get:

$$w^N f^N \left[\left[\left(\frac{\bar{\theta}}{\underline{\theta}^*} \right)^{\sigma-1} - 1 \right] + \chi^* W(.) \left(\frac{\bar{\theta}^S}{\underline{\theta}^*} \right)^{\sigma-1} \right] - \chi^* w^N f^N F(.) = \frac{(1-\lambda) w^N f_e}{1 - G(\underline{\theta}^*)}$$

with

$$\begin{aligned} W(w^N, w^S) &\equiv \left(\frac{w^N}{w^S} \right)^{(1-\eta)(\sigma-1)} - 1 \\ F(f^N, f^S, f^r) &\equiv \left(\frac{f^S + (1-\lambda) f^r}{f^N} \right) - 1 \end{aligned} \tag{60}$$

Solving for $\bar{\theta}$,

$$\Rightarrow \bar{\theta} = \left[\frac{(1-\lambda) f_e}{[1 - G(\underline{\theta}^*)] f^N} + \chi^* \left[F(.) - W(.) \left(\frac{\bar{\theta}^S}{\underline{\theta}^*} \right)^{\sigma-1} \right] + 1 \right]^{\frac{1}{\sigma-1}} \underline{\theta}^* \tag{61}$$

Number of active firms. Finally, I obtain the number of active firms, i.e. the number of final good producers, in the differentiated sector. For this, I consider as before:

$$H^* = \frac{(1-\gamma_0) E}{\bar{r}}$$

Using $\bar{r}$ from equation (57),

$$H^* = \frac{(1-\gamma_0) E}{\left[\left(\frac{\bar{\theta}}{\underline{\theta}^*} \right)^{\sigma-1} + \chi^* \left[\left(\frac{w^N}{w^S} \right)^{(\sigma-1)(1-\eta)} - 1 \right] \left(\frac{\bar{\theta}^S}{\underline{\theta}^*} \right)^{\sigma-1} \right] \sigma w^N f^N} \tag{62}$$

It is easy to see that when $w^N > w^S$, the number of active firms with tradable intermediate inputs is smaller that in the case when offshoring is not possible. This is an effect that comes from the reduction in the price index produced by offshoring firms, and thus leads to a stronger competition in the final good market.

C.4 Offshoring productivity cutoff

The firm at the offshoring productivity cutoff is indifferent between offshoring and domestic sourcing. Therefore,

$$\frac{\pi^S(\theta^{S,*})}{1-\lambda} - w^N f^r = \frac{\pi^N(\theta^{S,*})}{1-\lambda}$$

Therefore, the offshoring productivity cutoff is:

$$\theta^{S,*} = [(1-\gamma_0)E]^{\frac{\sigma}{1-\sigma}} Q \left[\frac{w^N \left[f^S - f^N + (1-\lambda) f^r \right]}{\psi^S - \psi^N} \right]^{\frac{1}{\sigma-1}} \tag{63}$$

Equivalently, the offshoring productivity cutoff can be expressed in terms of the market productivity cutoff:

$$\theta^{S,*} = \left(\frac{f^S + (1-\lambda) f^r}{f^N} - 1 \right)^{\frac{1}{\sigma-1}} \left[\left(\frac{w^N}{w^S} \right)^{(1-\eta)(\sigma-1)} - 1 \right]^{\frac{1}{1-\sigma}} \underline{\theta}^*$$

D Uncertainty - Dynamic model: Tradable intermediate inputs.

When a firm decides whether to explore her offshoring potential or remain active under domestic sourcing, she must compute the present value of the total offshoring profit premium that she expects to obtain, and compare it to the offshoring market research sunk cost f^r.

At time t, the present value of the expected offshoring profit premium for a firm with productivity θ, who is currently sourcing domestically, is given by:

$$\mathbb{E}_t \left[\Pi^{S,\text{prem}}(\theta) | f^S \leq f_t^S \right] = \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \lambda^{\tau-t} \pi_\tau^{S,\text{prem}} \left(\theta, f_\tau^S, Q(f_\tau^S), f^N, w^N, w^S \right) \Big| f^S \leq f_t^S \right]$$

From the equation above, it is clear that the expected profit premium flow depends on the expected offshoring fixed costs at the moment of the decision, and on the expected flow of new incoming information from the behaviour of other firms. The per period profits depend on the expected fixed costs at t and on the expected information flow. Therefore, they are affected by the effect that the increasing share of offshoring firms over time has in the sectoral price index, and thence in the sectoral aggregate consumption.

To simplify notation, I denote $\pi_t^{S,\text{prem}}(\theta, f_t^S, Q(f_t^S), f^N, w^N, w^S) \equiv \pi_t^{S,\text{prem}}(\theta)$, while $\pi^{S,prem}(\theta)$ refers to the per-period offshoring profit premium when there is no remaining uncertainty in the industry, i.e. when the true fixed cost has been revealed.

D.1 Proofs regarding Bayesian learning mechanism

After $t = 0$, firms sourcing domestically update their prior knowledge by observing the "physical state". By applying recursively Bayes rule, firms update every period their beliefs. The posterior distribution at time t is given by:

$$Y(f^S | f^S \leq f_t^S) = \frac{Y(f^S | f^S \leq f_{t-1}^S) Y(f_t^S | f^S)}{Y(f_t^S | f^S \leq f_{t-1}^S)}$$

where $Y(f^S|f^S \leq f^S_{t-1})$ indicates the prior distribution at time t, $Y(f^S_t|f^S)$ refers to the likelihood function, and the denominator is the scaling factor.

The likelihood takes the following form:

$$Y(f^S_t|f^S) = \begin{cases} 1 & \text{if } f_t \geq f^S \\ 0 & \text{if } f_t < f^S \end{cases}$$

Therefore, the posterior distribution is represented by:

$$Y(f^S|f^S \leq f^S_t) = \frac{Y(f^S|f^S \leq f^S_{t-1})}{Y(f^S_t|f^S \leq f^S_{t-1})}$$

which is similar to the learning mechanisms characterized by Rob (1991) and Segura-Cayuela and Vilarrubia (2008).

On the other hand, if a firm who explored offshoring in the period $t-1$ is doing domestic sourcing during period t, then this reveals that this firm has made a mistake. After paying the sunk cost, this firm learned that the true fixed cost in South is too high for her, i.e. she would obtain negative per-period offshoring profit premiums.

Therefore, given the assumption of a continuum of firms, this situation implies that the true fixed cost in South has been revealed and it corresponds to the maximum affordable fixed cost in South of the least productive firm doing offshoring in t.

As a summary, the knowledge that firms have before taking the offshoring decision in period t is given by:

$$f^S \sim \begin{cases} Y(f^S) & \text{with } f^S \in [\underline{f}^S, \bar{f}^S] \text{ for } t = 0 \\ Y(f^S|f^S \leq f^S_t) & \text{if } \tilde{f}^S_t = f^S_t < f^S_{t-1} \text{ for } t > 0 \\ f^S_t & \text{if } \tilde{f}^S_t < f^S_t \text{ for } t > 0 \end{cases} \tag{64}$$

D.2 Proof of the OSLA rule as optimal policy

The Bellman's equation takes the form:

$$\mathcal{V}_t(\theta;\theta_t) = \max\left\{V^o_t(\theta;\theta_t); \lambda_j \mathbb{E}_t\left[\mathcal{V}_{t+1}(\theta;\theta_{t+1})\right]\right\}$$

$$\mathcal{V}_t(\theta;\theta_t) = \max\left\{\mathbb{E}_t\left[\max\left\{0; \sum_{\tau=t}^{\infty} \lambda^{\tau-t}\pi^{S,prem}_\tau(\theta)\right\}\middle| f^S \leq f^S_t\right] - w^N f^r; \lambda \mathbb{E}_t\left[\mathcal{V}_{t+1}(\theta;\theta_{t+1})\right]\right\}$$

The goal is to find the optimal policy, which defines how many periods is optimal to wait given the information set at t.

$$a \in \arg\max_{a\in\{0,1\}} \mathcal{V}_t(\theta;\theta_t) = a\left[\mathbb{E}_t\left[\max\left\{0; \sum_{\tau=t}^{\infty} \lambda^{\tau-t}\pi^{S,prem}_\tau(\theta)\right\}\middle| f^S \leq f^S_t\right] - w^N f^r\right] + (1-a)\lambda \mathbb{E}_t\left[\mathcal{V}_{t+1}(\theta;\theta_{t+1}, a')\right]$$

where $a = 1$ denotes the action of trying offshoring in period t, while $a = 0$ refers to waiting.

Solution by policy iteration. By policy iteration, it is possible to prove that the One-Step-Look-Ahead (OSLA) rule is the optimal policy. In other words, that in expectation at t, waiting for one period dominates waiting for more periods.

At any given point in time, all the firms sourcing domestically have an expected flow of new information for every future period. According to this, the firms know they can obtain gains from waiting, by receiving new information and take the offshoring decision at a later period under a reduced uncertainty, or eventually with certainty if the true fixed cost has been revealed during the waiting period(s). However, the firms also face an opportunity cost of waiting, which is given by the offshoring profit premium that the firms can obtain by exploring the South in the current period and discovering their respective offshoring potential.

Let's define as $V_t^{w,1}(.), ..., V_t^{w,n}(.)$ the value of waiting in t for $1, ..., n$ periods, respectively.

$$\begin{aligned}
V_t^{w,1}(\theta;\theta_t,\theta_{t+1}) =& 0 + \frac{\left[Y(f_t^S) - Y(f_{t+1}^S)\right]}{Y(f_t^S)} \lambda \mathbb{E}_t \left[\max\left\{0; \frac{\pi^{S,prem}(\theta)}{1-\lambda} - w^N f^r \right\} \Big| f_{t+1}^S < f^S \leq f_t^S \right] \\
&+ \frac{Y(f_{t+1}^S)}{Y(f_t^S)} \lambda \left[\mathbb{E}_t \left[\max\left\{0; \sum_{\tau=t+1}^{\infty} \lambda^{\tau-t-1} \pi_\tau^{S,prem}(\theta) \right\} \Big| f^S \leq f_{t+1}^S \right] - w^N f^r \right] \\
V_t^{w,2}(\theta;\theta_t,\theta_{t+2}) =& 0 + \frac{\left[Y(f_t^S) - Y(f_{t+2}^S)\right]}{Y(f_t^S)} \lambda^2 \mathbb{E}_t \left[\max\left\{0; \frac{\pi^{S,prem}(\theta)}{1-\lambda} - w^N f^r \right\} \Big| f_{t+2}^S < f^S \leq f_t^S \right] \\
&+ \frac{Y(f_{t+2}^S)}{Y(f_t^S)} \lambda^2 \left[\mathbb{E}_t \left[\max\left\{0; \sum_{\tau=t+2}^{\infty} \lambda^{\tau-t-2} \pi_\tau^{S,prem}(\theta) \right\} \Big| f^S \leq f_{t+2}^S \right] - w^N f^r \right] \\
&\vdots \\
V_t^{w,n}(\theta;\theta_t,\theta_{t+n}) =& 0 + \frac{\left[Y(f_t^S) - Y(f_{t+n}^S)\right]}{Y(f_t^S)} \lambda^n \mathbb{E}_t \left[\max\left\{0; \frac{\pi^{S,prem}(\theta)}{1-\lambda} - w^N f^r \right\} \Big| f_{t+n}^S < f^S \leq f_t^S \right] \\
&+ \frac{Y(f_{t+n}^S)}{Y(f_t^S)} \lambda^n \left[\mathbb{E}_t \left[\max\left\{0; \sum_{\tau=t+n}^{\infty} \lambda^{\tau-t-n} \pi_\tau^{S,prem}(\theta) \right\} \Big| f^S \leq f_{t+n}^S \right] - w^N f^r \right]
\end{aligned}$$

It is straightforward to see:

$$\lim_{n\to\infty} V_t^{w,n}(\theta;\theta_t,\theta_n) = 0$$

The relevant analysis consists in the case when a firm θ faces a trade-off in her decision. This situation takes place when the value of offshoring for the firm θ in period t is non-negative, i.e. $V_t^o(\theta;.) \geq 0$, but nevertheless she can reduce the risk of exploring offshoring in t by waiting n periods for new incoming information[40]. In this situation, considering the decision characterised in section 2.2.3, the firm θ must decide what is the optimal number of periods for waiting and compare it to the value of offshoring in t in order to decide whether she will explore her offshoring potential or wait.

[40]Otherwise, the firms who have a negative value of offshoring in t, i.e. $V_t^o(\theta;.) < 0$, are not facing any trade-off in their decisions. In other words, they do not confront any dilemma, given that exploring their offshoring potential in t in not attractive, therefore they do not face any opportunity cost from waiting.

Therefore, if I narrow the analysis to the firms with a non-negative value of offshoring, i.e. $V_t^o(\theta;.) \geq 0$, thence it is easy to see that for each of these firms the value of waiting for any period $n = 1, ..., \infty$ is non-negative, i.e. $V_t^{w,n}(\theta;.) \geq 0 \forall n$.

So I will go one step further in analysing this trade-off situation, and define the number of periods that, in expectation at t, a firm θ finds optimal to wait. In this regard, following a similar argument as Segura-Cayuela and Vilarrubia (2008), I begin with the case of the marginal firm which compares the value of exploring offshoring now with the value of waiting for one period and explore in the next one, i.e. $\mathcal{D}_t(\theta;.) = V_t^0(\theta;.) - V_t^{w,1}(\theta;.) = 0$.

The argument of the proof is as follows. The value of waiting for n periods before exploring the offshoring potential falls at a rate of λ^n for firms that weakly prefer exploring the offshore' potential now than waiting for one period. Since $\lambda < 1$, waiting for any number of periods $n > 1$ is dominated by waiting for only one period. In other words, given Assumption A.6, if waiting for the information revealed in one period does not convince a firm to wait, waiting for two or more periods is less preferred, as the new information revealed every futher period is less. Therefore, to characterise the optimal equilibrium path it is only necessary to consider those firms who are deciding between exploring the offshoring potential in the current period or wait for one period.

I start by comparing the value of waiting for one period with the value of waiting for two periods, i.e. $V_t^{w,1}(\theta;.); V_t^{w,2}(\theta;.)$. As mentioned above, I focus the analysis in the marginal firm, i.e. the firm indifferent between explore offshoring today or wait for one period. Formally[41],

$$\begin{aligned}\mathcal{D}_t(\theta;\theta_t,\theta_{t+1}) &= V_t^0(\theta;\theta_t) - V_t^{w,1}(\theta;\theta_t,\theta_{t+1}) = 0\\&= \max\left\{0; \mathbb{E}_t\left[\pi_t^{S,prem}(\theta)\middle| f^S \leq f_t^S\right]\right\} - w^N f^r \left[1 - \lambda \frac{Y(f_{t+1}^S)}{Y(f_t^S)}\right]\\&+ \frac{[Y(f_t^S) - Y(f_{t+1}^S)]}{Y(f_t^S)} \lambda \mathbb{E}_t \left[\max\left\{0; \frac{\pi^{S,prem}(\theta)}{1-\lambda}\right\}\right.\\&\left.- \max\left\{0; \frac{\pi^{S,prem}(\theta)}{1-\lambda} - w^N f^r\right\} \middle| f_{t+1}^S < f^S \leq f_t^S\right] = 0\end{aligned}$$

Equivalently, the expression of the trade-off function for waiting for two peri-

[41]I show the derivation of the trade-off function in the main part of the paper, and the respective proofs are in Appendix D.3.

ods is given by:

$$\begin{aligned}\mathcal{D}_t(\theta;\theta_t,\theta_{t+2}) &= V_t^0(\theta;\theta_t) - V_t^{w,2}(\theta;\theta_t,\theta_{t+2})\\ &= \max\left\{0;\mathbb{E}_t\left[\pi_t^{S,prem}(\theta)+\lambda\pi_{t+1}^{S,prem}(\theta)\Big|f^S\leq f_t^S\right]\right\} - w^N f^r\left[1-\lambda^2\frac{Y(f_{t+2}^S)}{Y(f_t^S)}\right]\\ &+ \frac{[Y(f_t^S)-Y(f_{t+2}^S)]}{Y(f_t^S)}\lambda^2\mathbb{E}_t\left[\max\left\{0;\frac{\pi^{S,prem}(\theta)}{1-\lambda}\right\}\right.\\ &\left.-\max\left\{0;\frac{\pi^{S,prem}(\theta)}{1-\lambda}-w^N f^r\right\}\Big|f_{t+2}^S<f^S\leq f_t^S\right]\end{aligned}$$

I consider the case in which the third term of the RHS is zero for both trade-off functions[42]. Therefore, the trade-off functions become:

$$\begin{aligned}\mathcal{D}_t(\theta;\theta_t,\theta_{t+1}) &= \mathbb{E}_t\left[\pi_t^{S,prem}(\theta)\Big|f^S\leq f_t^S\right] - w^N f^r\left[1-\lambda\frac{Y(f_{t+1}^S)}{Y(f_t^S)}\right]\\ \mathcal{D}_t(\theta;\theta_t,\theta_{t+2}) &= \mathbb{E}_t\left[\pi_t^{S,prem}(\theta)+\lambda\pi_{t+1}^{S,prem}(\theta)\Big|f^S\leq f_t^S\right] - w^N f^r\left[1-\lambda^2\frac{Y(f_{t+2}^S)}{Y(f_t^S)}\right]\end{aligned}$$

If the value of waiting for one period dominates the value of waiting for two periods, thence:

$$\begin{aligned}&V_t^0(\theta;.) - V_t^{w,1}(\theta;.) - \left[V_t^0(\theta;.) - V_t^{w,2}(\theta;.)\right] \overset{!}{<} 0\\ \Leftrightarrow &V_t^{w,2}(\theta;.) - V_t^{w,1}(\theta;.) \overset{!}{<} 0\end{aligned}$$

By replacing with the respective trade-off functions in this last expression, I have:

$$\mathbb{E}_t\left[\pi_{t+1}^{S,prem}(\theta)\Big|f^S\leq f_t^S\right] \overset{!}{>} w^N f^r\left[\frac{Y(f_{t+1}^S)}{Y(f_t^S)} - \lambda\frac{Y(f_{t+2}^S)}{Y(f_t^S)}\right]$$

From the marginal firm condition above, I know:

$$\mathbb{E}_t\left[\pi_t^{S,prem}(\theta)\Big|f^S\leq f_t^S\right] = w^N f^r\left[1-\lambda\frac{Y(f_{t+1}^S)}{Y(f_t^S)}\right]$$

By Assumtion A.6,

$$1-\lambda Y\left(f_{t+1}^S|f^S\leq f_t^S\right) > Y\left(f_{t+1}^S|f^S\leq f_t^S\right) - \lambda Y\left(f_{t+2}^S|f^S\leq f_t^S\right)$$

and thus,

$$\begin{aligned}&\mathbb{E}_t\left[\pi_{t+1}^{S,prem}(\theta)\Big|f^S\leq f_t^S\right] > w^N f^r\left[\frac{Y(f_{t+1}^S)}{Y(f_t^S)} - \lambda\frac{Y(f_{t+2}^S)}{Y(f_t^S)}\right]\\ \Rightarrow &V_t^{w,2}(\theta;.) - V_t^{w,1}(\theta;.) < 0\end{aligned}$$

[42]This assumption allows me to focus in the most restrictive condition. It can be easily shown that if value of waiting for one period is optimal in this case, it is also optimal in the other cases.

From the result above, it is easy to see that $V_t^{w,n}(\theta;.) > V_t^{w,n+1}(\theta;.)$ for any period n. Therefore,

$$V_t^{w,1}(\theta;.) > V_t^{w,2}(\theta;.) > .... > V_t^{w,n}(\theta;.)$$

In other words, for those firms in a trade-off condition, in expectation at t, waiting for one period domintates waiting for longer periods.

Given that my interest concentrates in modelling the "offshoring vs. waiting" trade-off and characterising the decision rule that drives the movements of the offshoring productivity cutoff at every period t, I consider it is sufficient to focus on the case for which $V_t^o(\theta;.) \geq 0$, i.e. when firms face a non-negative value of offshoring[43].

Thus, using the result that OSLA is the optimal rule under this condition, the optimal value function takes the following expression:

$$\mathcal{V}_t(\theta;\theta_t) = \max\left\{\mathbb{E}_t\left[\max\left\{0;\sum_{\tau=t}^{\infty}\lambda^{\tau-t}\pi_\tau^{S,prem}(\theta)\right\}\Big| f^S \leq f_t^S\right] - w^N f^r; V_t^{w,1}(\theta;\theta_t,\theta_{t+1})\right\}$$

and by the transformation explained in section 2.2.3, I obtain the trade-off function.

D.3 Derivation of the trade-off function

$$\mathcal{D}_t(\theta;\theta_t,\theta_{t+1}) = V_t^o(\theta;\theta_t,\theta_{t+1}) - V_t^{w,1}(\theta;\theta_t,\theta_{t+1})$$

Decomposing the value of offshoring,

$$\begin{aligned} V_t^o(\theta;.) =& \max\left\{0;\mathbb{E}_t\left[\pi_t^{S,prem}(\theta)\Big| f^S \leq f_t^S\right]\right\} - w^N f^r \\ &+ \frac{[Y(f_t^S) - Y(f_{t+1}^S)]}{Y(f_t^S)}\lambda\mathbb{E}_t\left[\max\left\{0;\frac{\pi^{S,prem}(\theta)}{1-\lambda}\right\}\Big| f_{t+1}^S < f^S \leq f_t^S\right] \\ &+ \frac{Y(f_{t+1}^S)}{Y(f_t^S)}\lambda\mathbb{E}_t\left[\max\left\{0;\sum_{\tau=t+1}^{\infty}\lambda^{\tau-t-1}\pi_\tau^{S,prem}(\theta)\right\}\Big| f^S \leq f_{t+1}^S\right] \end{aligned}$$

Note that $\frac{[Y(f_t^S)-Y(f_{t+1}^S)]}{Y(f_t^S)}$ denotes the probability that the true fixed cost is revealed in period t, while $\frac{Y(f_{t+1}^S)}{Y(f_t^S)}$ is the probability that the true value is not revealed but the uncertainty will reduce given the new information flow.

[43] I show here that there is no degeneration in firms' choices when $V_t^o(\theta;.) < 0$. In other words, I show that there is no reversion of the trade-off function sign under this situation, so firms will never find optimal to explore offshoring in t when $V_t^o(\theta;.) < 0$. If $V_t^{w,n}(\theta;.) \geq 0$, then the trade-off function $\mathcal{D}(\theta;.)$ is negative for any waiting period n with a positive value of waiting.

On the other hand, from a first sight it is possible to think that if $V_t^{w,n}(\theta;.) < 0$ this may result in a positive value for the trade-off function $\mathcal{D}(\theta;.)$. It is easy to see that in these cases $|V_t^o(\theta;.)| > |V_t^{w,n}(\theta;.)|$. Therefore, the trade-off function is still negative in all those cases.

In consequence, when the value of offshoring in t is negative, the trade-off function leads to a waiting decision. However, the number of periods that these firms find optimal to wait depends on the productivity level of each of them. Sufficiently low productive firms, for which $V_t^{w,n}(\theta;.) < 0 \forall n$, find optimal to wait infinite periods. On the other hand, firms relatively more productive that the previous ones find optimal to wait a finite number of periods, which is decreasing in the productivity of the firms.

Going one step further, by introducing the maximum affordable fixed cost of production in South for the firm, i.e. $f^S(\theta)$,

$$\begin{aligned} V_t^o(\theta;.) =& \max\left\{0;\mathbb{E}_t\left[\pi_t^{S,prem}(\theta)\Big|f^S\leq f_t^S\right]\right\}-w^Nf^r \\ &+\frac{[Y(f_t^S)-Y(f^S(\theta))]}{Y(f_t^S)}\lambda 0 \\ &+\frac{[Y(f^S(\theta))-Y(f_{t+1}^S)]}{Y(f_t^S)}\lambda\mathbb{E}_t\left[\frac{\pi^{S,prem}(\theta)}{1-\lambda}\Big|f_{t+1}^S<f^S\leq f^S(\theta)\right] \\ &+\frac{Y(f_{t+1}^S)}{Y(f_t^S)}\lambda\mathbb{E}_t\left[\max\left\{0;\sum_{\tau=t+1}^{\infty}\lambda^{\tau-t-1}\pi_{j,\tau}^{S,prem}(\theta)\right\}\Big|f^S\leq f_{t+1}^S\right] \end{aligned}$$

The probability of true value revealed and above the maximum affordable fixed cost for the firm θ is $\frac{[Y(f_t^S)-Y(f^S(\theta))]}{Y(f_t^S)}$, and the probability of the fixed cost revealed below it is $\frac{[Y(f^S(\theta))-Y(f_{t+1}^S)]}{Y(f_t^S)}$.

$$\begin{aligned} \Rightarrow V_t^o(\theta;.) =& \max\left\{0;\mathbb{E}_t\left[\pi_t^{S,prem}(\theta)\Big|f^S\leq f_t^S\right]\right\}-w^Nf^r \\ &+\frac{[Y(f^S(\theta))-Y(f_{t+1}^S)]}{Y(f_t^S)}\lambda\mathbb{E}_t\left[\frac{\pi^{S,prem}(\theta)}{1-\lambda}\Big|f_{t+1}^S<f^S\leq f^S(\theta)\right] \\ &+\frac{Y(f_{t+1}^S)}{Y(f_t^S)}\lambda\mathbb{E}_t\left[\sum_{\tau=t+1}^{\infty}\lambda^{\tau-t-1}\pi_{\tau}^{S,prem}(\theta)\Big|f^S\leq f_{t+1}^S\right] \end{aligned}$$

On the other hand, with an equivalent decomposition for the value of waiting one period,

$$\begin{aligned} V_t^{w,1}(\theta;.) =& 0+\frac{\left[Y(f_t^S)-Y(f_{t+1}^S)\right]}{Y(f_t^S)}\lambda\mathbb{E}_t\left[\max\left\{0;\frac{\pi^{S,prem}(\theta)}{1-\lambda}-w^Nf^r\right\}\Big|f_{t+1}^S<f^S\leq f_t^S\right] \\ &+\frac{Y(f_{t+1}^S)}{Y(f_t^S)}\lambda\left[\mathbb{E}_t\left[\max\left\{0;\sum_{\tau=t+1}^{\infty}\lambda^{\tau-t-1}\pi_{\tau}^{S,prem}(\theta)\right\}\Big|f^S\leq f_{t+1}^S\right]-w^Nf^r\right] \end{aligned}$$

$$\begin{aligned} V_t^{w,1}(\theta;.) =& 0+\frac{\left[Y(f_t^S)-Y(f^S(\theta))\right]}{Y(f_t^S)}\lambda 0+\frac{\left[Y(f^S(\theta))-Y(f_{t+1}^S)\right]}{Y(f_t^S)} \\ &\times\lambda\mathbb{E}_t\left[\max\left\{0;\frac{\pi^{S,prem}(\theta)}{1-\lambda}-w^Nf^r\right\}\Big|f_{t+1}^S<f^S\leq f_t^S\right] \\ &+\frac{Y(f_{t+1}^S)}{Y(f_t^S)}\lambda\left[\mathbb{E}_t\left[\sum_{\tau=t+1}^{\infty}\lambda^{\tau-t-1}\pi_{\tau}^{S,prem}(\theta)\Big|f^S\leq f_{t+1}^S\right]-w^Nf^r\right] \end{aligned}$$

$$\begin{aligned} \Rightarrow V_t^{w,1}(\theta;.) =& \frac{\left[Y(f^S(\theta))-Y(f_{t+1}^S)\right]}{Y(f_t^S)}\lambda \\ &\times\mathbb{E}_t\left[\max\left\{0;\frac{\pi^{S,prem}(\theta)}{1-\lambda}-w^Nf^r\right\}\Big|f_{t+1}^S<f^S\leq f^S(\theta)\right] \\ &+\frac{Y(f_{t+1}^S)}{Y(f_t^S)}\lambda\left[\mathbb{E}_t\left[\sum_{\tau=t+1}^{\infty}\lambda^{\tau-t-1}\pi_{\tau}^{S,prem}(\theta)\Big|f^S\leq f_{t+1}^S\right]-w^Nf^r\right] \end{aligned}$$

Replacing the value of offshoring and the value of waiting for one period in the trade off function gives the following equivalent expressions,

$$\begin{aligned}\mathcal{D}_t(\theta;.) = \max\left\{0; \mathbb{E}_t\left[\pi_t^{S,prem}(\theta)\middle| f^S \le f_t^S\right]\right\} - w^N f^r\left[1 - \lambda\frac{Y(f_{t+1}^S)}{Y(f_t^S)}\right] \\ + \frac{[Y(f_t^S) - Y(f_{t+1}^S)]}{Y(f_t^S)}\lambda\mathbb{E}_t\left[\max\left\{0; \frac{\pi^{S,prem}(\theta)}{1-\lambda}\right\}\right. \\ \left. - \max\left\{0; \frac{\pi^{S,prem}(\theta)}{1-\lambda} - w^N f^r\right\}\middle| f_{t+1}^S < f^S \le f_t^S\right]\end{aligned} \tag{65}$$

$$\begin{aligned}\mathcal{D}_t(\theta;.) = \max\left\{0; \mathbb{E}_t\left[\pi_t^{S,prem}(\theta)\middle| f^S \le f_t^S\right]\right\} - w^N f^r\left[1 - \lambda\frac{Y(f_{t+1}^S)}{Y(f_t^S)}\right] \\ + \frac{[Y(f(\theta)) - Y(f_{t+1}^S)]}{Y(f_t^S)}\lambda\mathbb{E}_t\left[\frac{\pi^{S,prem}(\theta)}{1-\lambda}\right. \\ \left. - \max\left\{0; \frac{\pi^{S,prem}(\theta)}{1-\lambda} - w^N f^r\right\}\middle| f_{t+1}^S < f^S \le f^S(\theta)\right]\end{aligned} \tag{66}$$

Proposition 1 implies that the probability of the true value being revealed below the maximum affordable fixed cost for the firm θ is zero. If it is not zero, this means that a firm with a lower productivity (i.e. $\tilde{\theta}_{t+1} < \theta$) has tried offshoring before the firm θ, which is not possible due to Proposition 1. In other words, given the sequential shape of the offshoring equilibrium path, led by the most productive firms in the market, a firm θ will discover her positive offshoring potential by waiting with probability zero.

Therefore, the trade off function becomes:

$$\mathcal{D}_t(\theta;\theta_t,\theta_{t+1}) = \max\left\{0; \mathbb{E}_t\left[\pi_t^{S,prem}(\theta)\middle| f^S \le f_t^S\right]\right\} - w^N f^r\left[1 - \lambda\frac{Y(f_{t+1}^S)}{Y(f_t^S)}\right]$$

D.4 Proof of Proposition 1

From section 2.1, it is clear that the offshoring profit premium $\pi^{S,prem}(\theta)$ is increasing in θ.

Thence, taking the trade off function expression from equation (65), it is straightforward to see that $\frac{\partial\mathcal{D}_t(\theta;\theta_t,\tilde{\theta}_{t+1})}{\partial\theta} \ge 0$.

D.5 Proof of Proposition 2

$$\mathcal{D}_t(\tilde{\theta}_{t+1};\tilde{\theta}_t,\tilde{\theta}_{t+1}) = 0$$

$$\mathbb{E}_t[\pi_t^{S,prem}(\tilde{\theta}_{t+1})|f^S \le f_t^S] - w^N f^r\left[1 - \lambda\frac{Y(\tilde{f}_{t+1}^S)}{Y(f_t^S)}\right] = 0$$

Replacing $\pi_t^{S,prem}(\tilde{\theta}_{t+1})$ with expressions for $\pi_t^S(\tilde{\theta}_{t+1})$ and $\pi_t^N(\tilde{\theta}_{t+1})$ from equation (22),

$$\tilde{\theta}_{t+1}^{\sigma-1}[(1-\gamma_0)E]^{\sigma}\tilde{Q}_{t+1}^{1-\sigma}[\psi^S-\psi^N] = w^N\left[E_t(f^S|f^S\leq f_t^S) - f^N + f^r\left(1-\lambda\frac{Y(\tilde{f}_{t+1}^S)}{Y(f_t^S)}\right)\right]$$

$$\tilde{\theta}_{t+1} = [(1-\gamma_0)E]^{\frac{\sigma}{1-\sigma}}\tilde{Q}_{t+1}\left[\frac{w^N\left[E_t(f^S|f^S\leq f_t^S) - f^N + f^r\left(1-\lambda\frac{Y(\tilde{f}_{t+1}^S)}{Y(f_t^S)}\right)\right]}{\psi^S-\psi^N}\right]^{\frac{1}{\sigma-1}}$$

D.6 Proofs of Propositions 3, 4 (long-run properties)

By *Assumption A.7*,

$$\mathcal{D}_t(\bar{\bar{\theta}};\bar{\bar{\theta}},\bar{\bar{\theta}}) > 0$$

$$\mathbb{E}_t[\pi_t^{S,prem}(\bar{\bar{\theta}})|f^S\leq \bar{f}^S] - w^N f^r(1-\lambda) > 0$$

$$\frac{r_t^{N,*}(\bar{\bar{\theta}})}{\sigma}W(.) - w^N E_t(f^S|f^S\leq \bar{f}^S) - w^N[f^r(1-\lambda) - f^N] > 0$$

Taking the limit of the trade off function as $t\to\infty$,

$$\mathcal{D}(\theta_\infty;\theta_\infty,\theta_\infty) = \frac{r^{N,*}(\theta_\infty)}{\sigma}W(.) - w^N E\left(f^S|f^S\leq f_\infty^S\right) - w^N\left[f^r(1-\lambda) - f^N\right]$$

Totally differentiating $\mathcal{D}(\theta_\infty;\theta_\infty,\theta_\infty)$ with respect to each of its arguments:

$$\frac{d\mathcal{D}(\theta_\infty;\theta_\infty,\theta_\infty)}{d\theta_\infty} = \frac{W(.)}{\sigma}\frac{\partial r^{N,*}(\theta_\infty)}{\partial\theta_\infty} - w^N\frac{\partial E(f^S|f^S\leq f_\infty^S)}{\partial f_\infty^S}\frac{\partial f_\infty^S}{\partial\theta_\infty}$$

By equation (11), f_∞^S is given by:

$$f_\infty^S \equiv f^S(\theta_\infty) = \frac{r^N(\theta_\infty)}{\sigma w^N}\left[\left(\frac{w^N}{w^S}\right)^{(1-\eta)(\sigma-1)} - 1\right] + f^N$$

Therefore,

$$\begin{aligned}\frac{d\mathcal{D}(\theta_\infty;\theta_\infty,\theta_\infty)}{d\theta_\infty} &= \frac{W(.)}{\sigma}\frac{dr^{N,*}(\theta_\infty)}{d\theta_\infty} - w^N\frac{W(.)}{w^N\sigma}\frac{dr^{N,*}(\theta_\infty)}{d\theta_\infty}\frac{\partial E(f^S|f^S\leq f_\infty^S)}{\partial f_\infty^S}\\ &= \frac{dr^{N,*}(\theta_\infty)}{d\theta_\infty}\frac{W(.)}{\sigma}\left[1 - \frac{\partial E(f^S|f^S\leq f_\infty^S)}{\partial f_\infty^S}\right]\end{aligned}$$

From this expression, $\frac{dr^{N,*}(\theta_\infty)}{d\theta_\infty} > 0$ and $\frac{W(.)}{\sigma} > 0$.

By *Assumption A.6*,

$$\frac{\partial[f_t^S - E(f^S|f^S\leq f_t^S)]}{\partial f_t^S} > 0 \Rightarrow 1 - \frac{\partial E(f^S|f^S\leq f_t^S)}{\partial f_t^S} > 0$$

$$\Rightarrow \frac{\partial E(f^S|f^S\leq f_t^S)}{\partial f_t^S} < 1$$

Thence, using this assumption, the expression in brackets

$$\left[1 - \frac{\partial E(f^S | f^S \leq f^S_\infty)}{\partial f^S_\infty}\right] > 0$$

Only in the limit, when the distribution collapses with the lower bound,

$$\frac{\partial E(f^S | f^S \leq f^S_t)}{\partial f^S_t} = 1 \Rightarrow \mathcal{D}(\theta_\infty; \theta_\infty, \theta_\infty) = 0$$

Therefore, it is possible to see that this problem has at most one unique fixed point. Therefore, the fixed point defined in Proposition 3 is unique.

E Uncertainty - Multicountry model.

Let's consider the difference between offshoring profit premium with perfect information between firms sourcing from South and from East. For a firm with productivity θ, it is given by:

$$\begin{aligned}\pi^{S,prem}(\theta) - \pi^{E,prem}(\theta) =& \frac{r^N(\theta)}{\sigma}(w^N)^{(1-\eta)(\sigma-1)}\left[\frac{(w^E)^{(1-\eta)(\sigma-1)} - (w^S)^{(1-\eta)(\sigma-1)}}{(w^E w^S)^{(1-\eta)(\sigma-1)}}\right] \\ &- w^N\left[f^S - f^E\right]\end{aligned}$$

Under uncertainty, this expression for a firm θ currently sourcing in East in period t is given by:

$$\begin{aligned}\mathbb{E}_t[\pi^{S,prem}(\theta) | f^S \leq f^S_t] - \pi^{E,prem}_t(\theta) =& \frac{r^N(\theta, Q_t)}{\sigma}(w^N)^{(1-\eta)(\sigma-1)} \\ &\times \left[\frac{(w^E)^{(1-\eta)(\sigma-1)} - (w^S)^{(1-\eta)(\sigma-1)}}{(w^E w^S)^{(1-\eta)(\sigma-1)}}\right] \\ &- w^N\left[\mathbb{E}_t(f^S | f^S \leq f^S_t) - f^E\right]\end{aligned}$$

Contents